VOLUME 103 PART A NUMBER 11 OCTOBER 1956

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OF

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## PART A

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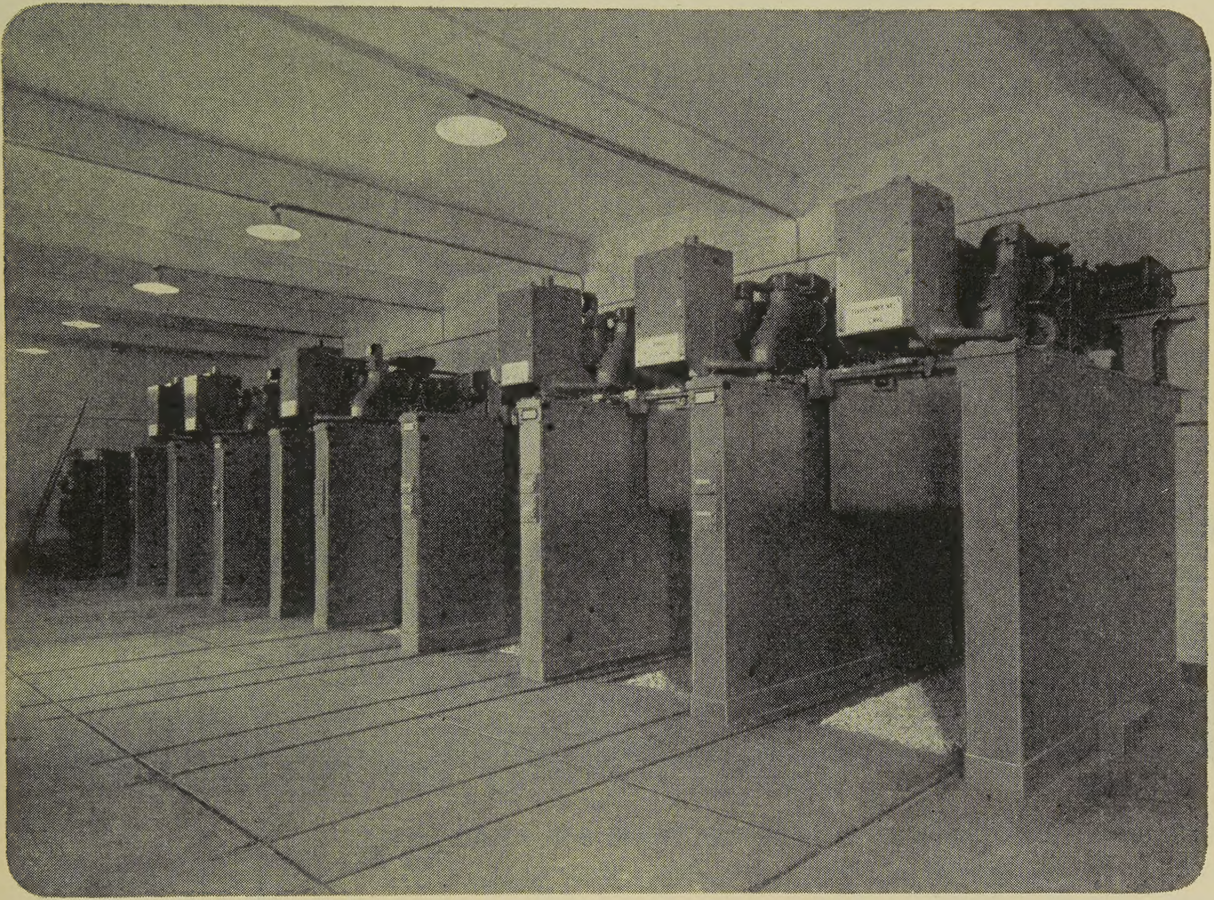

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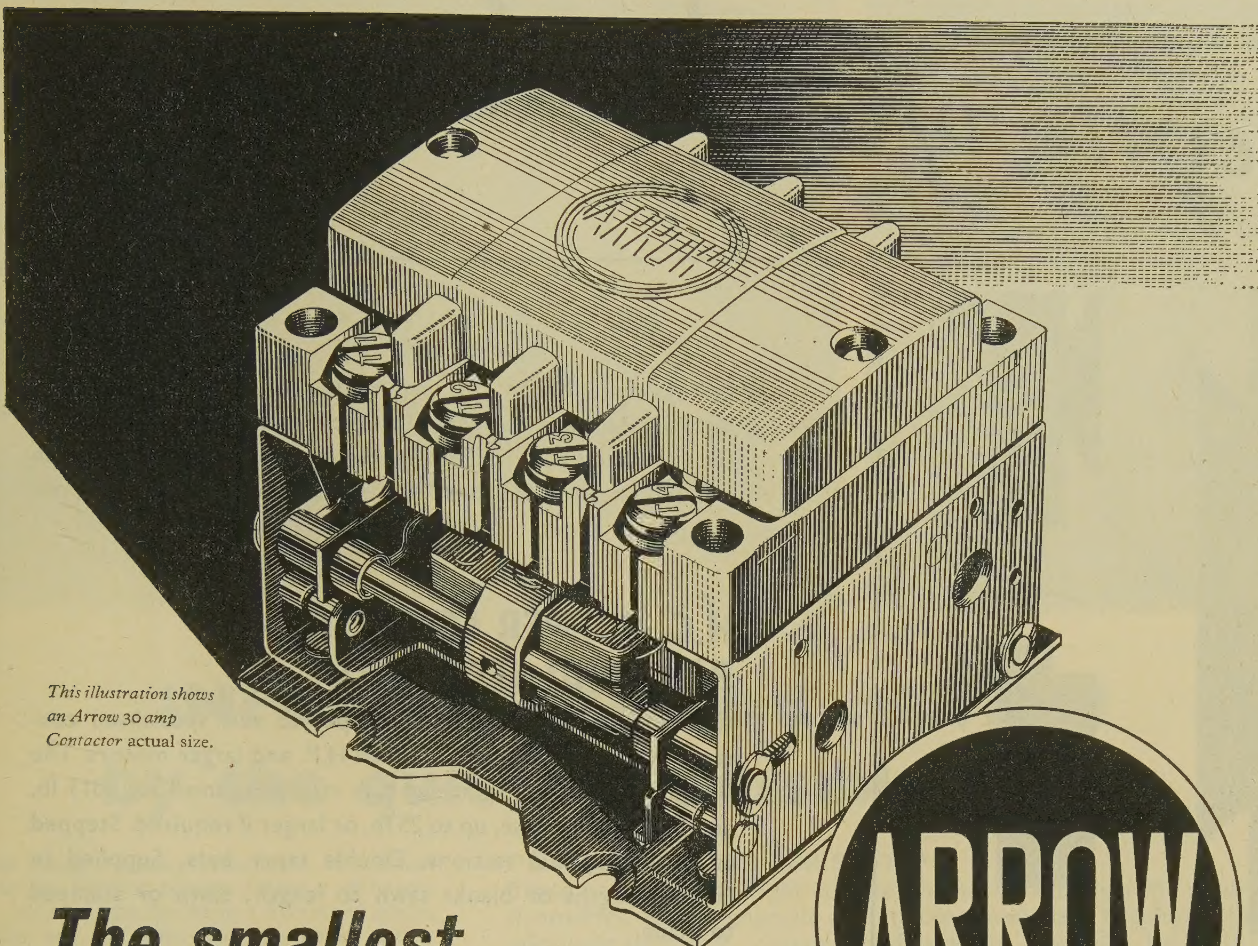

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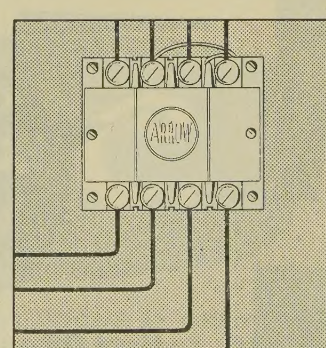


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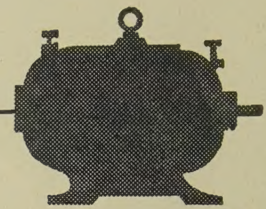

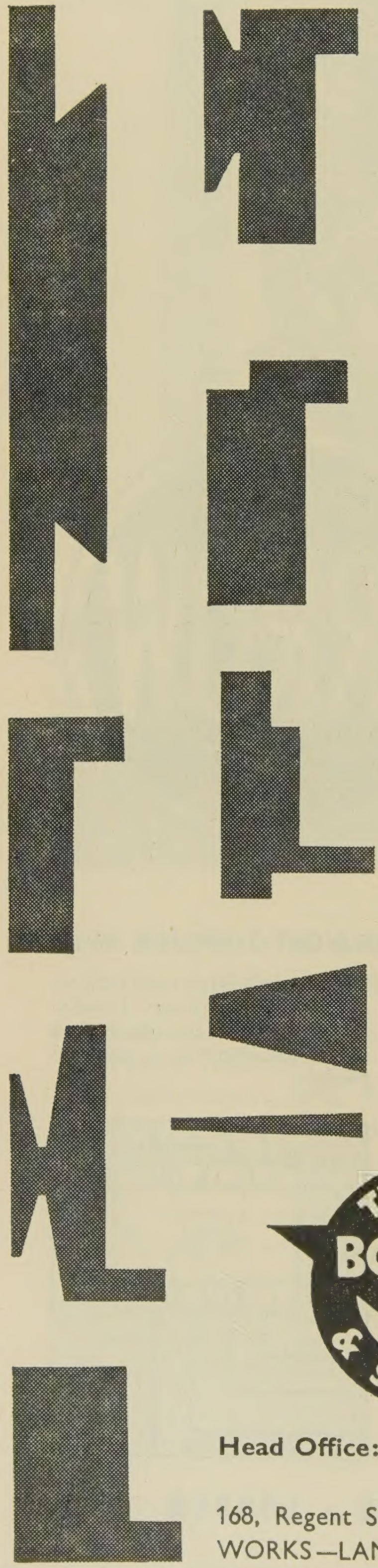

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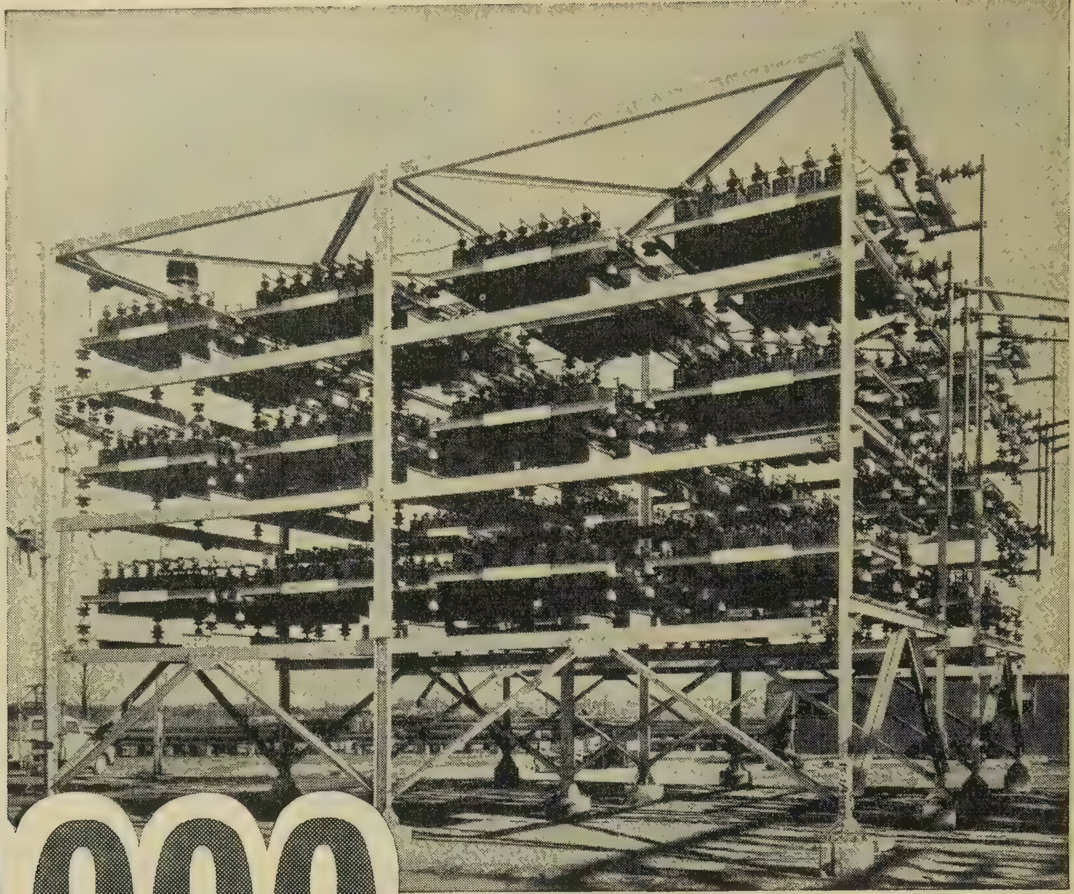

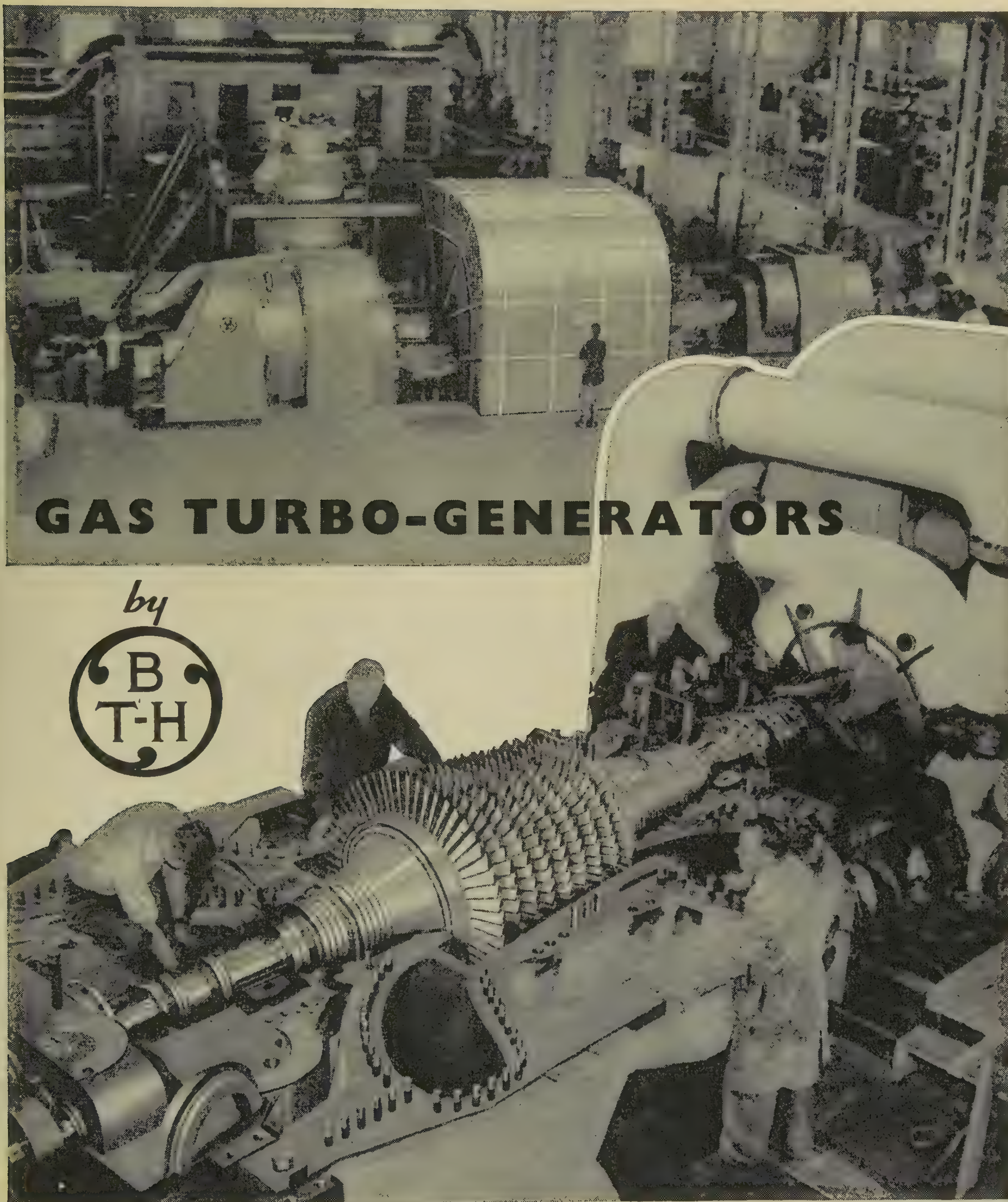


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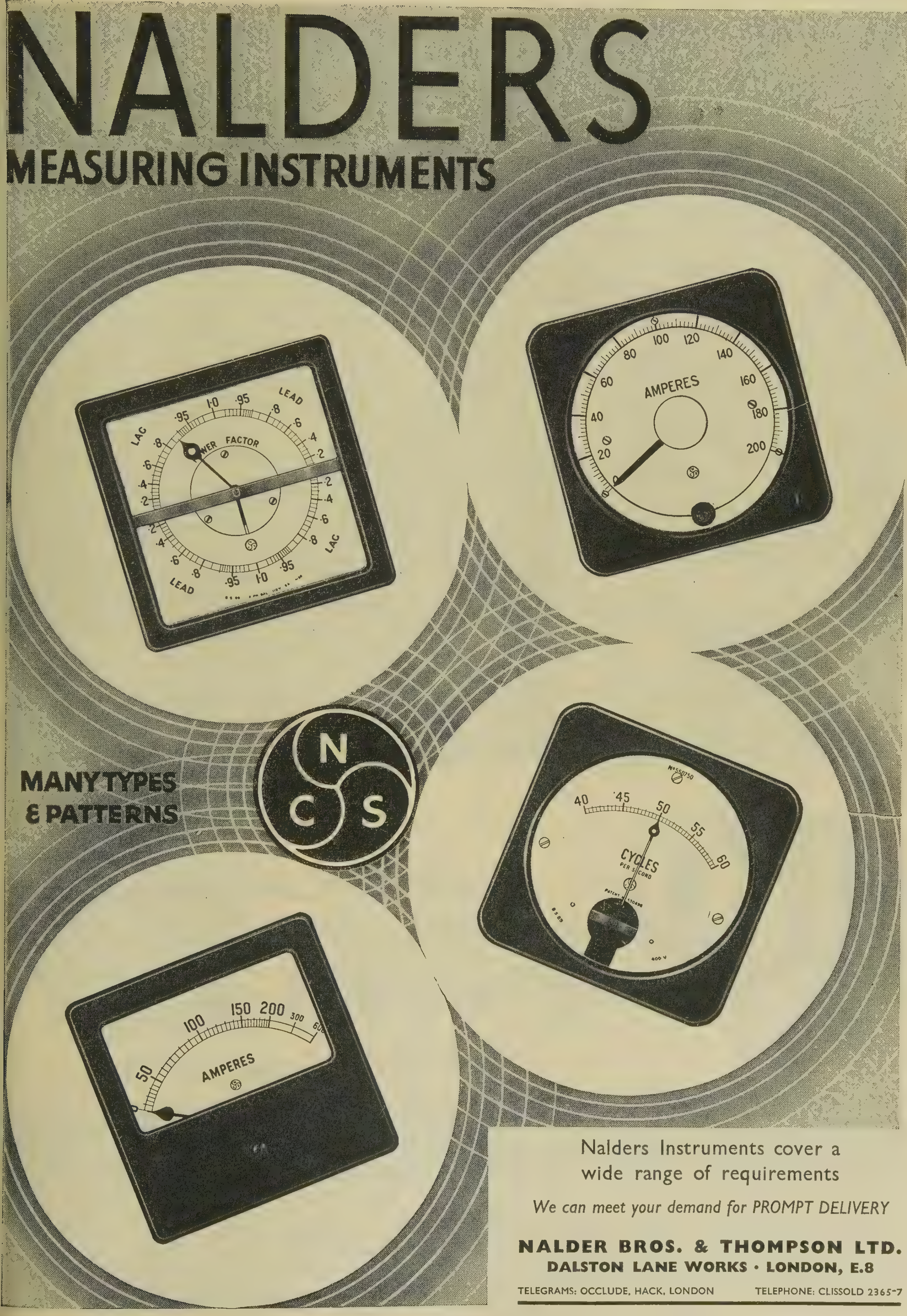
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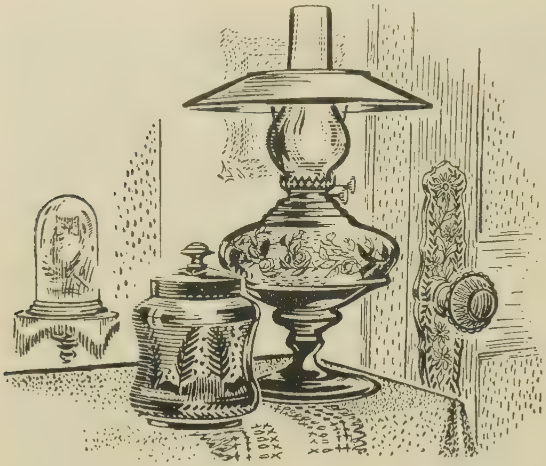



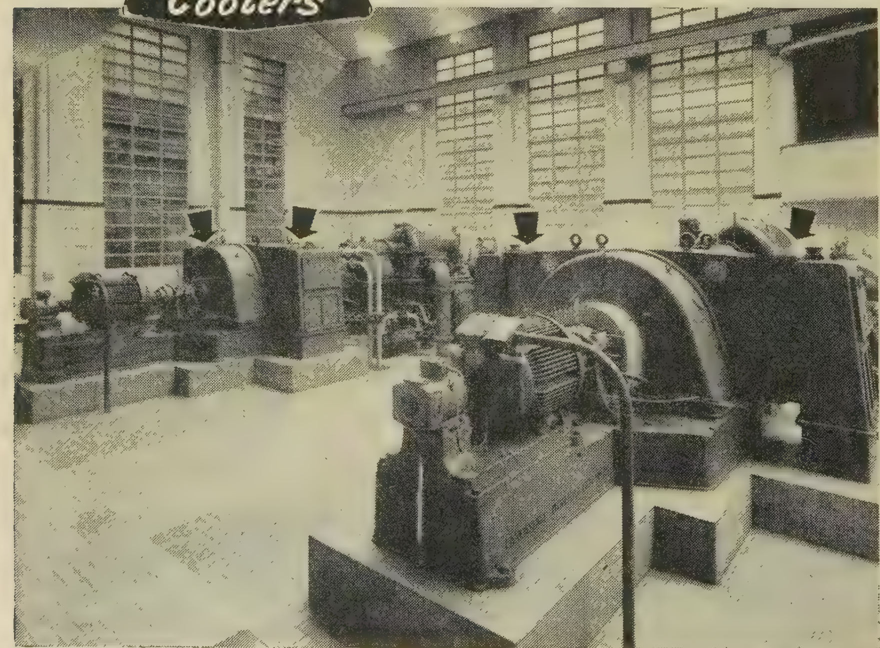

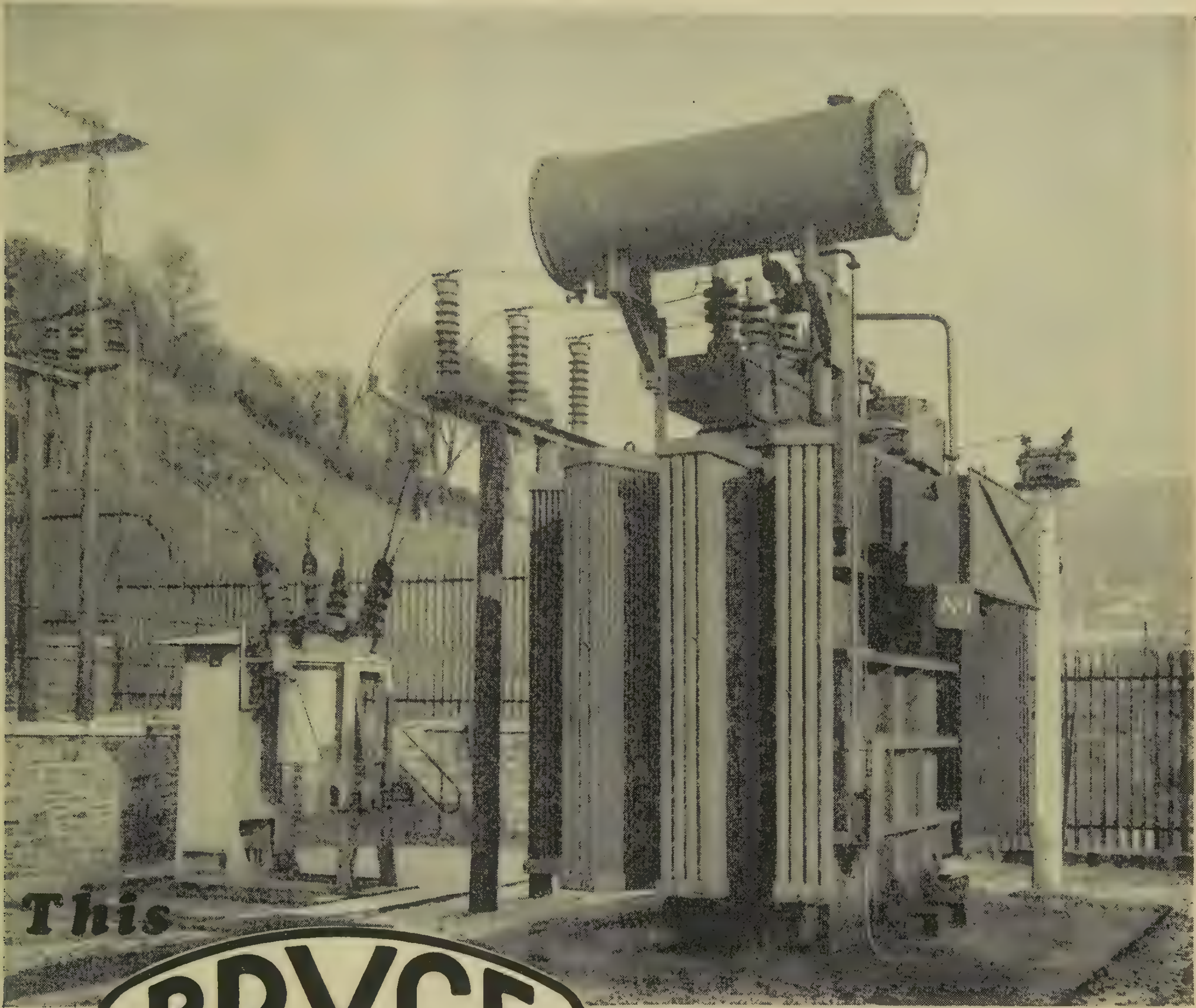

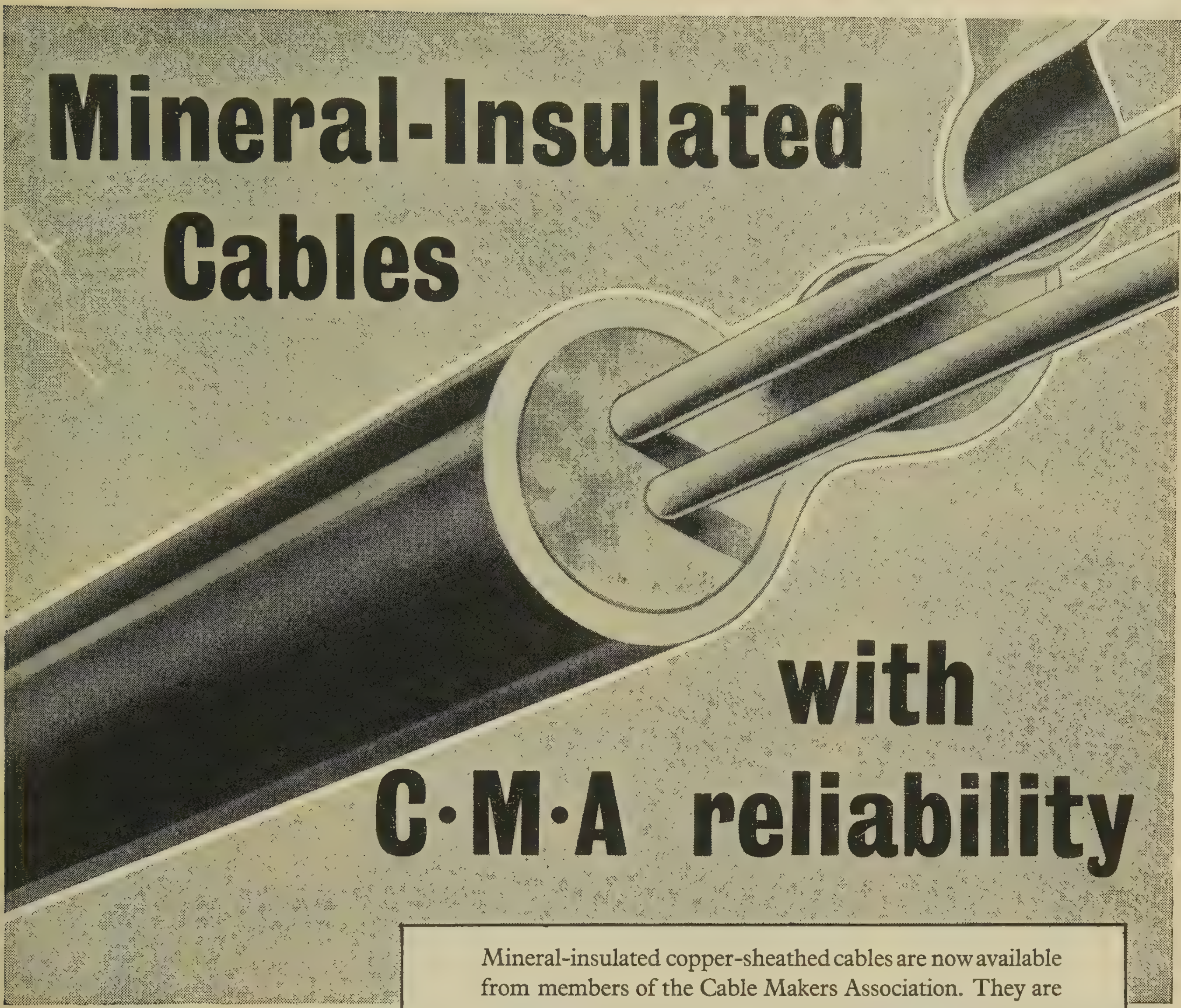


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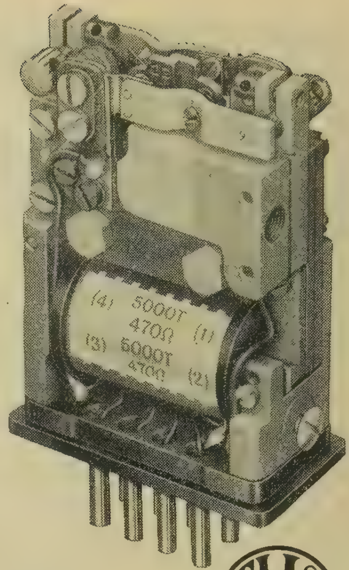


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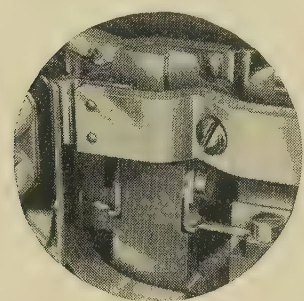

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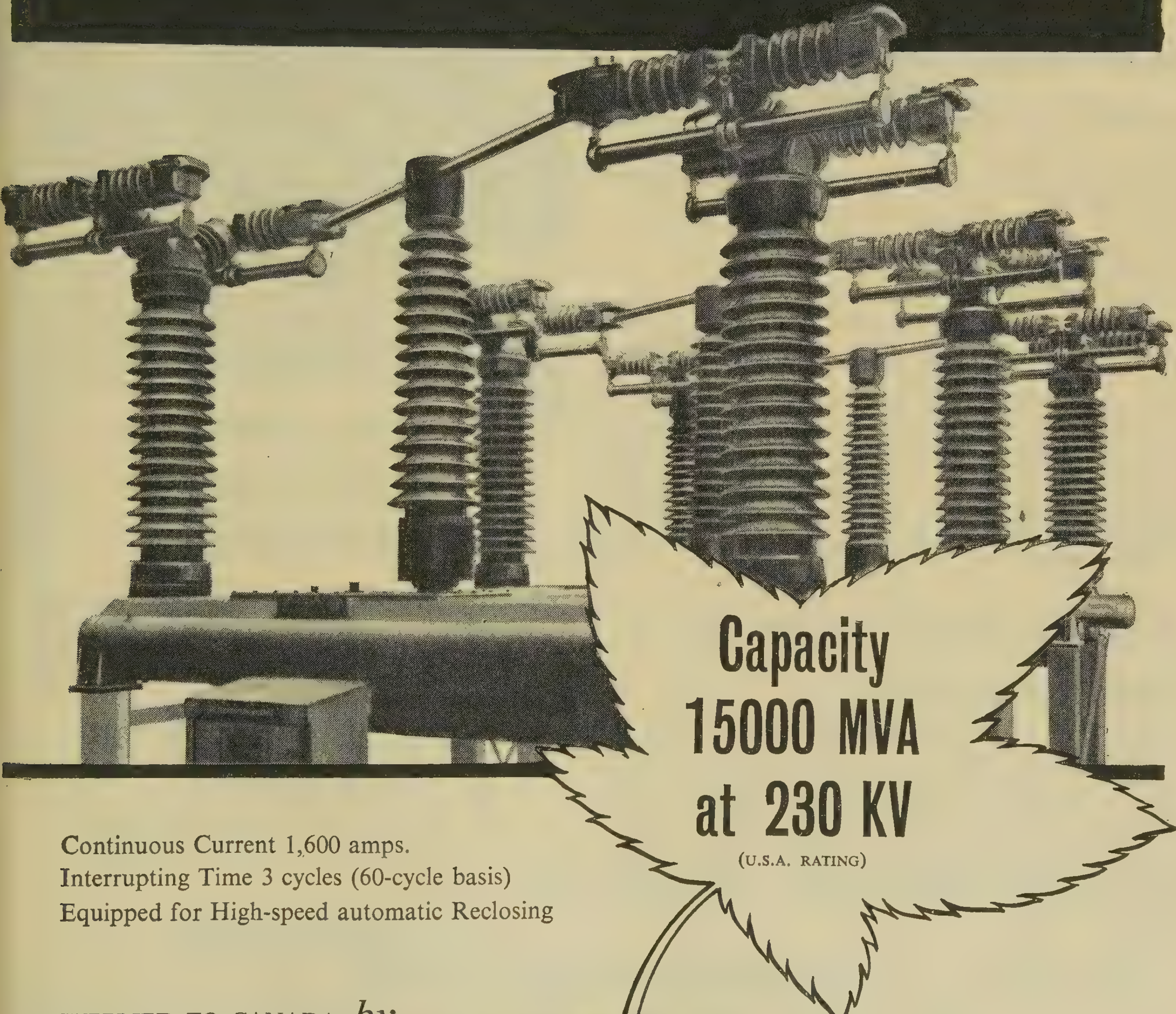


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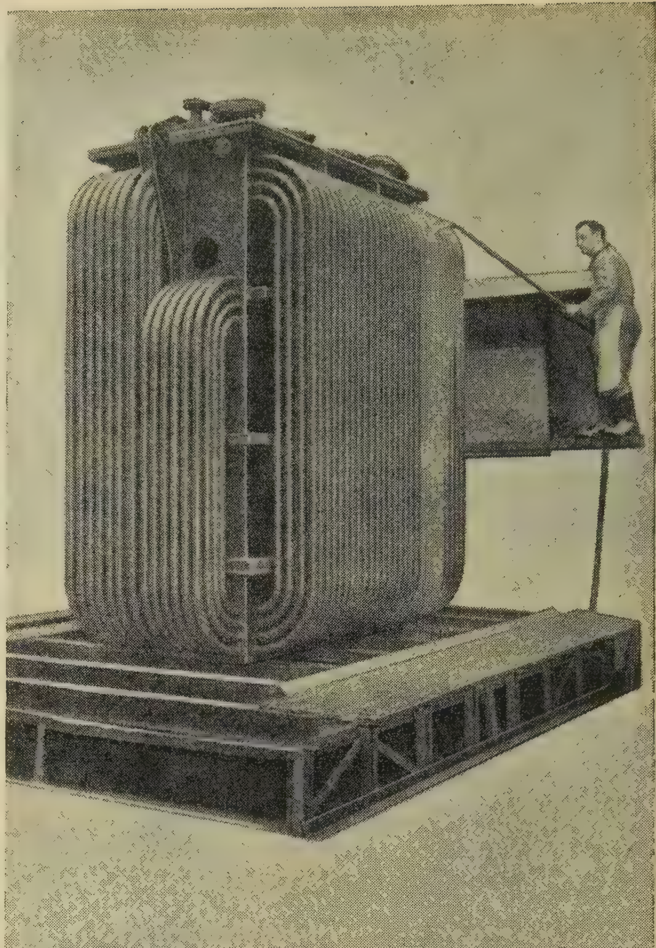

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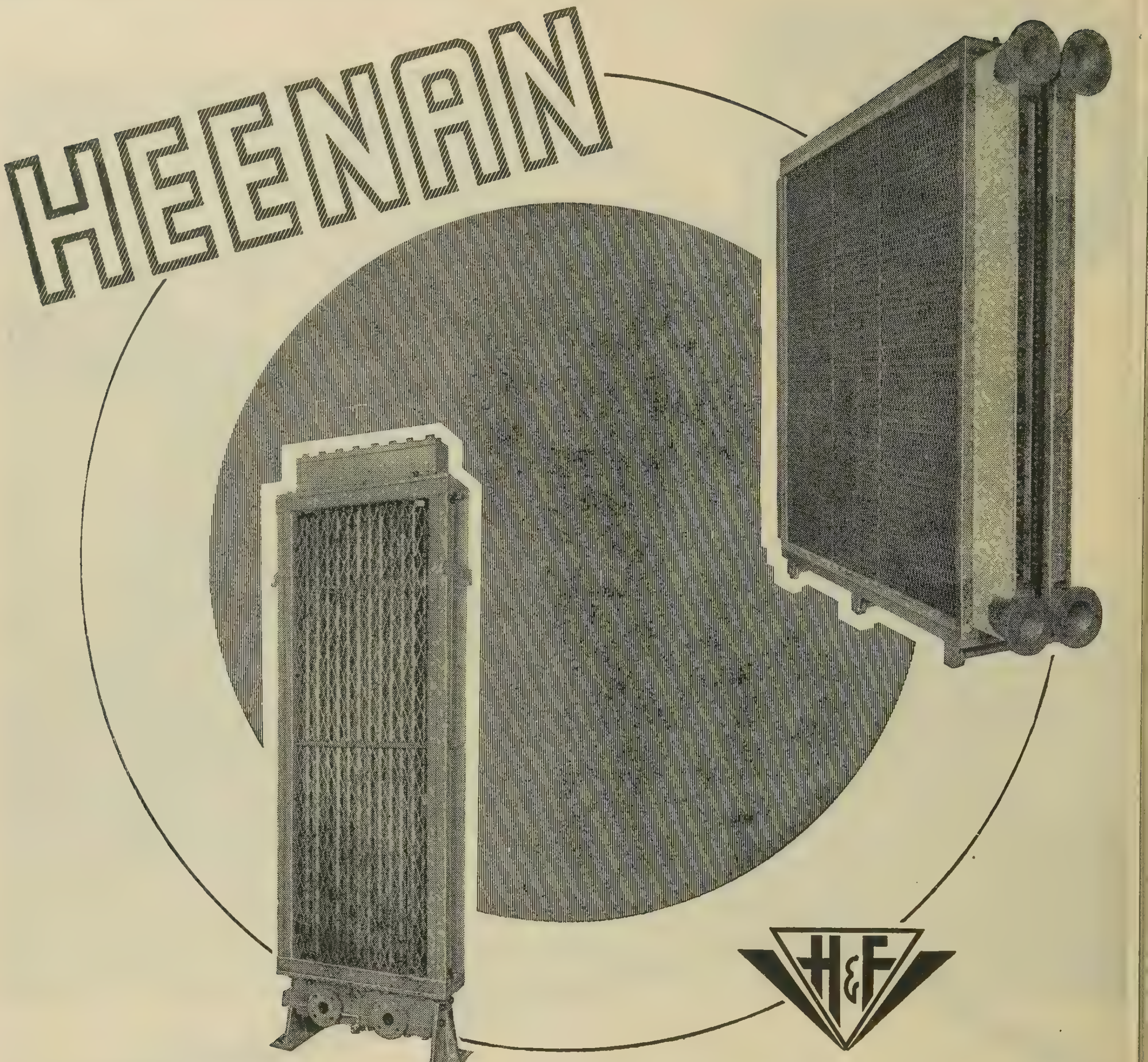
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EDITED UNDER THE SUPERINTENDENCE OF W. K. BRASHER, C.B.E., M.A., M.I.E.E., SECRETARY

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Paper No. 1942 M
Dec. 1955

# AN EVALUATION OF TWO RAPID METHODS OF ASSESSING THE THERMAL RESISTIVITY OF SOIL

By M. W. MAKOWSKI, Dipl.Ing., and K. MOCHLINSKI, Dipl.Ing., Associate Members.

(*The paper was first received* 1*st April, and in revised form* 25*th August*, 1955. *It was published in December*, 1955, *and was read before a Joint Meeting of the* MEASUREMENT AND CONTROL SECTION *and the* SUPPLY SECTION 28*th February*, 1956.)

## SUMMARY

The paper evaluates two methods recently proposed for the rapid assessment of the thermal resistivity of soil *in situ*, with special application to the assessment of cable ratings.

Brief consideration is given to the factors determining the thermal resistivity of the soil, i.e. its composition, compactness and moisture content, and then to the present standard method of measurement by buried spherical or cylindrical heater.

The basis of the so-called transient-needle method is then set out. Theory, errors due to departure from ideal conditions and practical application are considered.

A method based on soil sampling is next discussed. After critical considerations of the underlying theoretical and practical work by Gemant, Kersten and Mickley in the United States, a nomogram is introduced by means of which the thermal resistivity of sand–clay mixtures of known physical characteristics may be simply determined. The practical application of the two rapid methods to field measurements is described, and results obtained by the various methods are compared.

Finally, the new methods are discussed and compared with that using the static buried heater.

## LIST OF PRINCIPAL SYMBOLS

$\theta$ = Temperature.
$t$ = Time.
$\rho$ = Density.
$\rho_d$ = Dry density.
$c$ = Specific heat.
$r$ = Distance from centre of needle.
$q$ = Heat output per unit length of heater.
$I$ = Current through needle.
$R$ = Total resistance of the heater circuit.
$l$ = Length of needle.
$D = \frac{k}{c\rho}$ = Thermal diffusivity.
$g = 1/k$ = Thermal resistivity.
$k = 1/g$ = Thermal conductivity.
$k_s$ = Thermal conductivity of solid soil.
$k_w$ = Thermal conductivity of water.
$p$ = Percentage of clay in clay–sand mixtures of soil.
$m$ = Moisture content (weight of water as a percentage of weight of dry soil).
$W_1$ = Weight of container filled with moist soil.
$W_2$ = Weight of container filled with dry soil.
$W_3$ = Weight of container only.
$V$ = Volume of container.

The paper is based on Draft Report Ref. F/T182 of the British Electrical and Allied Industries Research Association.

## (1) INTRODUCTION

The thermal properties of the soil are important in a wide range of technical problems. The thermal resistivity of the soil largely determines the continuous current-carrying capacity of buried cables, and, with thermal diffusivity, their peak-load rating.[1] It determines the kilowatt rating of a system of buried pipes forming a part of a heat pump using the earth as a source of low-grade heat, and the additional thermal insulation required to reduce to an acceptable value the heat losses from buried pipes carrying heated liquids or steam.

The thermal resistivity of the soil often varies considerably even within one locality; it can, however, be said to depend mainly on three factors:

(*a*) *Composition of the Soil.*—In the majority of cases soil is a mixture of constituents commonly known as gravel, coarse and fine sand, silt, loam and clay. Chalk and peat are often encountered, as also are waste industrial materials. The thermal resistivities of the base materials of which the particles are composed vary considerably.

(*b*) *Compactness of the Soil.*—Compactness is defined by the dry density, i.e. the weight of unit volume of undisturbed soil less the weight of moisture contained in that volume. For most soils the dry density may be assumed constant, but with heavy clays (more than 30% clay) it will vary with the moisture content. The thermal resistivity of soil of given composition decreases with increase in dry density.

(*c*) *Moisture Content.*—The moisture content is likely to vary throughout the year; thus the thermal resistivity of soil is also subject to variation, and a measured value of thermal resistivity should be referred to the moisture content when the measurement was taken. The thermal resistivity of soil of given composition and dry density decreases with increase in moisture content.

Unless extreme precautions are taken, determination of the

thermal resistivity of a sample of soil by laboratory measurement of heat flow is subject to considerable error, owing to the difficulty of preserving or reproducing the original structure, and measurement of the thermal resistivity *in situ* is to be preferred. The generally accepted method[1,2,3] is to place in the ground to be investigated a metal container such as a sphere or cylinder with internal heater, to apply heat at a known constant rate, and to calculate the thermal resistivity from the rate of heat input, the measured final steady-state temperature rise of the container or of a part of the soil near it, and the geometry of the system. Whilst, by the use of suitable boring tools, the spheres or cylinders may be inserted with minimum disturbance of the soil, sufficient time must be allowed for the soil to settle to its original condition before heating is started. A steady temperature is attained after 5–7 days from the start of the heating. If accurate results are to be obtained, a total of several weeks must elapse between the placing of the sphere and the availability of first results.

Although the sphere method is accurate and specially valuable for long-term testing, a reasonably complete survey of any track requires a fairly large number of spheres with the associated equipment for constant power supply. In practice there is a need for a quicker and less expensive method. Two rapid methods tested by E.R.A. have recently been described.[3] The present paper gives the evidence on which the recommendation of these methods is based. They can be used separately according to the local conditions, or together, with a correlation of the results obtained with the corresponding moisture content of the soil.

It must be emphasized that in some soils the thermal resistivity of the soil in the immediate vicinity of a hot body may be increased above that of the surrounding soil by movement of moisture away from the heated body. Neither of the methods discussed takes any account of this effect. The sphere method if properly used can give some indication of it, but this cannot at present be related to the performance expected with buried cables. The effect is not thought to have affected in any important way the operation of buried cables under the conditions in which they were generally used in Great Britain up to 1954, but with a general adoption of high operating temperatures in cables, some allowance should be made for this effect, particularly with cables laid in fairly porous soils likely to be heavily loaded during the summer and early autumn.

## (2) RAPID METHODS AVAILABLE

The possibility of determining the thermal resistivity from measurements made during the transient stage of heating was investigated by the E.R.A. in 1937–38, and it was then proved by the principle of similitude that if a heater of any convenient shape is used first in soil of known thermal resistivity and diffusivity and then in the soil to be investigated, the values of the thermal resistivity and diffusivity of the unknown soil can be found from the displacement of the heating curves. The method was tested, but certain difficulties in its use were found.

Other workers have concentrated on the use of a line-source heater, which has the advantage that its theory leads to absolute rather than comparative results.[4] Since the 1939-45 War, further work has been done, mainly in Canada, on the practical application of this theory to the measurement of the thermal resistivity of soil. Several papers[5–11] give a broad description of the development and practical utilization of the laws of transient heat-flow from a line source for determination of the thermal resistivity of soil, and of solids and liquids in general.

The agreement found in practice between this method and the steady-state method is sufficient to allow the new method to be used with reasonable confidence.

The second method is the assessment of the thermal resistivity from knowledge of the soil structure as observed on a suitably selected sample. The theoretical treatments by Gemant[12,13] and Mickley[14] and practical formulae by Kersten,[15] enabling the thermal resistivity of soil to be assessed from its constitution, have hitherto been worked out only for mixtures of clay and sand. They do not give concordant results in all circumstances, but those of Gemant and Kersten show satisfactory agreement with experiment in all the cases in which comparison has so far been possible. The method here proposed is based on the theoretical work of Gemant and the experiments of Kersten.

## (3) TRANSIENT-NEEDLE METHOD

### (3.1) Theory of Operation

The transient needle is a portable instrument consisting of an electrically heated probe containing a temperature-measuring element, and it is constructed to represent as closely as necessary or practicable an infinite line-source of heat.

If in a medium of thermal diffusivity $D$ the temperature is a function only of the time $t$ and of the distance $r$ from the linear source, the relation between the variables is given by the Fourier equation in cylindrical co-ordinates,

$$\frac{\partial\theta}{\partial t} = D\left(\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r}\right)$$

Starting from thermal equilibrium with constant rate of heat production during time $t$ the solution

$$\theta = \frac{gq}{4\pi}\left[-Ei\left(-\frac{r^2}{4Dt}\right)\right]$$

can be obtained, in which $Ei$ is the symbol of the exponential integral function.

This formula is exact for a probe of infinite length and radius $r$, of material of the same diffusivity and conductivity as the soil in which it is inserted, and having an infinitesimally thin central heater-wire, provided that the temperature is measured on the surface of the probe and that there is no contact resistance at the boundary between the probe and the external medium. Then, if $r^2/4Dt$ is sufficiently small, i.e. if, for given values of $r$ and $D$, $t$ is sufficiently large,

$$g = \frac{4\pi}{q}\frac{\theta_2 - \theta_1}{\log t_2/t_1}$$

This means that if heat is applied at a line source and the temperature $\theta$ at a small radius $r$ is plotted against the logarithm of the time which elapses from the moment of load application the graph should become a straight line, the slope of which directly indicates the thermal resistivity $g$ of the medium.

Fig. 1 shows theoretical curves of temperature against time measured ($a$) at a distance $r = \sqrt{0{\cdot}02D}$ from a line source ($b$) for the same line source at a distance $r = \sqrt{2D}$ and ($c$) at the surface of a cylindrical needle of negligible thermal capacity and radius $r = \sqrt{2D}$. The linear portion of curve ($b$) is approached within 3% at the time $4r^2/D$, i.e. for the measurement at radius 0·2 cm in soil of $D = 0{\cdot}02\,\mathrm{cm^2/sec}$, after 8 sec.

With practical constructions the needle will be larger and will not have the same thermal characteristics as the soil, and the times required are greater.

Typical transient curves taken with needles of different characteristics and in low- and high-resistivity soils are shown in Fig. 2. In practical calculations only a portion of the straight line of temperature rise across one decade of the logarithmic time scale is read from the graph and converted to temperature by the use

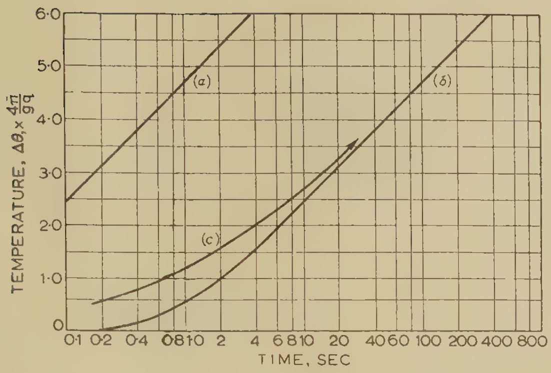


**Fig. 1.**—Theoretical transient heating at radius $r$ in medium of diffusivity $D$ for linear and cylindrical sources.

(a) Line source; measurement at $r = \sqrt{(0 \cdot 02D)}$.
(b) Line source; measurement at $r = \sqrt{(2D)}$.
(c) Cylindrical source of radius $r = \sqrt{(2D)}$; measurement at surface.

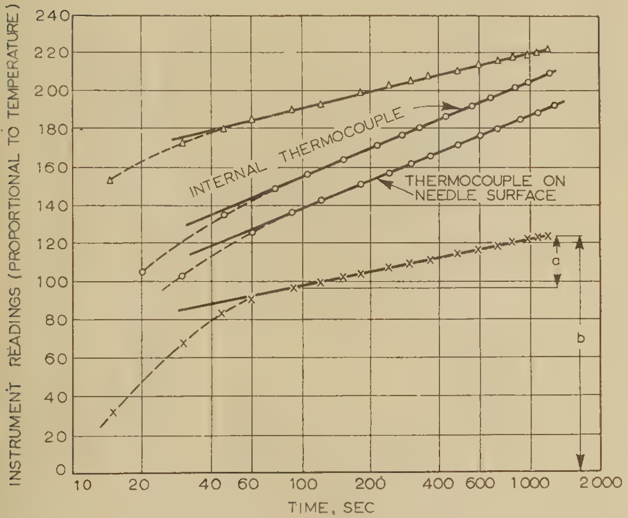


**Fig. 2.**—Transient heating curves.

○—$\frac{1}{4}$ in (outside diameter) brass needle in high-resistivity sand (150 °C-cm/W); loading 0·45 W/cm; approximate maximum temperature 65° C.
△—$\frac{3}{16}$ in (outside diameter) Duralumin needle in low-resistivity clay (60 °C-cm/W); loading 0·27 W/cm; approximate maximum temperature 35° C.
×—$\frac{1}{4}$ in outside diameter aluminium needle in low-resistivity clay (60 °C-cm/W); loading 0·21 W/cm.

(a) Useful portion of temperature rise.
(b) Total temperature rise proportional to $I^2$.

of an appropriate calibration chart. A practical formula of the following form can then be used:

$$g = \left(13 \cdot 85 \frac{l}{R}\right) \frac{\Delta\theta}{I^2} \text{ °C-cm/watt (thermal ohm-cm)}$$

where $\Delta\theta$ is the temperature rise in degrees centigrade during one logarithmic decade of time, $I$ is in amperes, $l$ in inches and $R$ in ohms.

### (3.2) Errors

With an infinitely long needle the errors due to the fact that the needle does not represent a true line source, can, in principle, be made negligible by continuing observations over a sufficiently long time. This is so because whatever the nature and material of the probe the rate of heat retention within it is a continuously decreasing fraction of the constant rate of heat production, so that the probe tends to represent a uniform cylindrical source of heat, for which the curve of temperature against the logarithm of time must, as with a line source, eventually become linear, with a slope proportional to the thermal resistivity of the medium.

With a probe of a finite length an error must arise from the end concentration of heat-flow lines. Thus the results obtained from the measurement at the centre of the needle are affected according to the length and longitudinal thermal conductivity of the probe. Generally, with the ratio length/diameter of the order of 100/1 these errors are negligible.

Two difficulties of a practical nature prevent the attainment of accuracy by measurement over very long times. The first is that with a probe of finite length, whatever its cross-section, the final temperature rise with constant heat output is finite; thus although the rate of temperature rise at the centre is initially almost identical with that of an finite probe, it cannot continue so indefinitely. Limitations in this respect have not been fully explored, but they have not been found important in practice so long as the probe follows other design conditions. A more important limitation arises from the fact that, since the total permissible temperature rise is usually limited by the properties of the medium under test, if measurements are continued for too long the actual changes in temperature measured become very small and commensurate with those due to unavoidable changes in loss of heat from the head of the probe, to changes in the surface temperatures of the soil with sun, rain, etc., and to small changes in the rate of power input to the heater. It is this last aspect, together with considerations of convenience and of the precise location of the sample tested, which mainly compels the use of rather small probes for soil surveys.

Where it is particularly important that measurements be made within a short time of the commencement of heating, e.g. where the temperature range of interest is small and very close to the initial temperature, or where it is desired to measure the thermal conductivity of liquids before they are seriously affected by convection currents, artifices may be used, consisting essentially of iteration processes, by means of which the earlier portions of the $\theta/\log t$ curves may be made to contribute towards the assessment of thermal resistivity.[5,6,8,9] In general, however, these are not appropriate for rapid field testing, particularly where anomalous results must be checked as they arise.

Not all factors contributing to errors are easily represented mathematically, and derived compensation may sometimes lead to over-correction. A further, more involved, theoretical development is claimed to be useful when the probe radius cannot be kept small and when there is a definite contact resistance between the probe and the surrounding medium.[16,17,18]

Errors may arise if the output of the heater is not kept constant and if the times and temperatures are not recorded accurately, since the linear portion of the temperature rise may be small compared with the total. If the proportion is 1 : $n$ an error of $\delta$% in the nominal current during the linear portion of the rise creates an error in resistivity assessment of $2\delta n$%. This effect is particularly important when the temperature-measuring element is inside the tube and the total temperature observed is the sum of the temperature at the tube surface and the drop through part of the thermal resistance (which may be of low heat capacity) between the heater and the tube wall.

Errors may also arise from non-uniformity and disturbance of the soil. Those due to non-uniformity can perhaps hardly be classed as errors, since it is inherent in a measurement of the kind in question that the result obtained is an average property of the sample tested. But if the soil tested is stratified the results obtained will represent the properties of the soil at the centre of the needle, weighted by those of the soil at the ends to an extent depending in part on the longitudinal thermal conductivity of the needle. Measurements obtained in non-homogeneous soil do not form a perfect straight-line graph.

In inserting the needle, whether directly or into a pilot hole, the soil will be slightly disturbed and slightly compressed. The importance of this factor is difficult to assess and probably will vary according to the nature of the soil. The effects of disturbance may be expected to be greatest in a friable or stony soil; more than one test should then be made and only the lower values of resistivity taken as indicating the properties of the soil.

### (3.3) Practical Application

#### (3.3.1) The Needle.

The needle probe is constructed in the form of a long thin tube and should have low heat capacity and high thermal diffusivity. In order to minimize the effect of the finite length of the probe its length should be about 100 times its external diameter. The diameter of the probe is dictated, however, by the space needed by the heater and temperature-measuring elements, and also, for field measurements, by the need for robustness. Although needles of diameter as low as 0·03 in have been used for the testing of liquids[11] an external diameter of $\frac{3}{16}-\frac{1}{4}$ in has been found to be the best compromise for measurement on soil. Larger probes, although they may be more robust, are more difficult to insert in the soil, take a longer time before reaching the linear portion of the $\theta/\log t$ relation and are more liable to errors.

Needle probes as designed and used by E.R.A. and based initially on that described by Hooper and Lepper,[8] are shown in Figs. 3 and 4. Fig. 5 is a diagram of connections of the needle

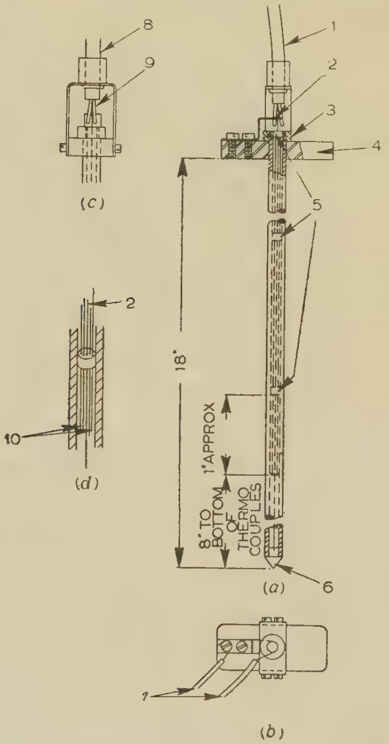


**Fig. 3.**—Constructional details of a 3/16 in Duralumin needle.

(a) Assembly.
(b) End view.
(c) Side view showing bracket.
(d) Enlarged view of spacer and thermocouples.

(1) Plastic tube protecting thermocouples.
(2) Heater wire (No. 30 s.w.g.) soft-soldered to clip.
(3) Brass collar pressed on Duralumin tube.
(4) Keramot base cemented to Duralumin tube.
(5) Keramot spacers.
(6) Steel tip.
(7) Heater connections joined to clip and brass collar.
(8) Keramot bushing.
(9) Separate insulation for thermocouples.
(10) Two enamelled, silk-covered copper-Eureka (No. 33–35 s.w.g.) thermocouples.

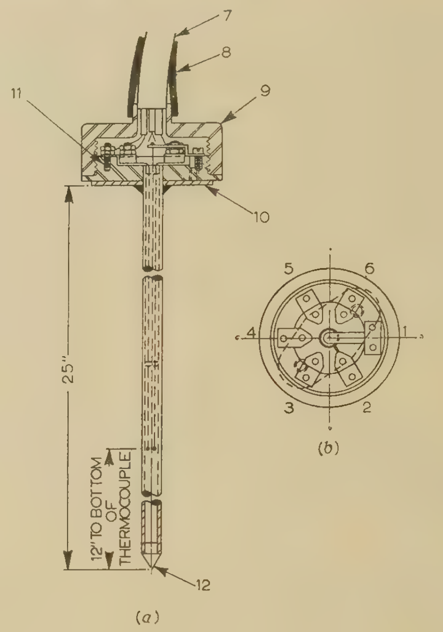


**Fig. 4.**—$\frac{1}{4}$ in brass needle with detachable reference junctions.

(a) Assembly.
(b) Plan of base, showing terminal connections.

(1) Phosphor-bronze clip to heater wire.
(2) Copper tag to heater return.
(3) and (5) Copper tags for thermocouples.
(4) and (6) Eureka tags and washers for thermocouples.
(7) Plastic protecting tube.
(8) Rubber tubing.
(9) Resin-bonded cambric base and cover.
(10) Brass base.
(11) Studs sunk in resin cement.
(12) Brass tip.

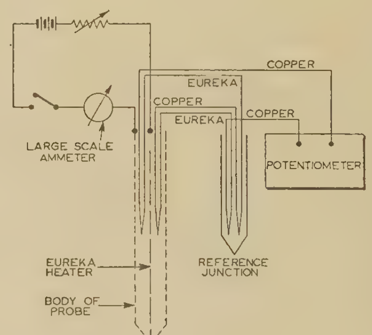


**Fig. 5.**—Diagram of needle-probe (Fig. 2) circuits using a thermopile of two copper-Eureka thermocouples.

probe, which for temperature measurements has a thermopile composed of two thermocouples.

#### (3.3.2) The Heating Circuit.

The heater, designed to generate heat uniformly along the tube, may consist of a straight wire with the return circuit through the outer tube, or a double spiral, or a loop. To avoid change of dissipation with change of temperature, the heater is made of material having a very low temperature-coefficient of resistance such as Eureka. For the same reason stability of current is of great importance. The heater must be able to dissipate about 0·5–0·8 watt/cm, according to the desired range of temperature rise. Power is usually supplied from batteries.

#### (3.3.3) Temperature Measurement.

The temperature measurement is usually made by means of a fine well-insulated wire thermocouple placed near the middle of

the needle length, either singly or forming a thermopile (as shown in Fig. 5) to produce a higher potential and to avoid small anomalies in temperature distribution. Radial position of the thermocouples within the tube does not in principle affect the results obtained. They are, however, so positioned that the indicated temperature-rise does not include a substantial fraction due to the internal thermal resistance of the needle. Thermocouples must be well protected from leakage current. The reference junctions may be mounted inside a small metal tube, e.g. 0·5 in outside diameter, suitable for insertion into the soil to serve as a temperature reference level.

When temperature is measured by means of a resistance thermometer precautions must be taken to avoid error due to the end effect or to uneven internal thermal properties along the length of the needle due, for example, to a bulky temperature-measuring coil. The temperature-measuring circuit must be separate from the heater circuit for the reason stated.

(3.3.4) **Testing.**

The measurement must be made at points free of surface water, preferably on a dry day. At a test point, top soil is first removed with a spade or an earth borer. The needle is then inserted as far as the depth of interest. Usually, a hole must first be made by means of a guided steel rod of diameter not greater than that of the needle. As soon as the needle reaches the temperature of the surrounding soil (about 10 min) the heater may be energized by a predetermined current and the temperature measured thereafter at suitable time intervals (Fig. 2).

## (4) ASSESSMENT FROM CONSTITUTION OF SOIL

### (4.1) Theory

(4.1.1) **General Considerations.**

Several investigators have proposed theoretical and empirical methods of assessing the thermal conductivity of soil indirectly. These methods are compared and analysed in this Section from the point of view of practicability and agreement with experimental results. Only sand–clay mixtures may be treated by the proposed rapid method; it has not yet been possible to treat chalky, rocky and made-up soils or soils containing a large proportion of humus.

It is assumed that soil particles may be classified in three groups according to linear dimensions: (i) pebbles (more than 2 mm); (ii) sand (2–0·002 mm); (iii) clay (less than 0·002 mm). At depths at which cables are laid organic content is usually small and may be neglected. Only fine soil having particles with dimensions less than 2 mm is taken into account. The presence of pebbles will generally increase the thermal conductivity.

The dry density $\rho_d$ of the soil is taken as a basis, i.e. the weight of unit volume of undisturbed soil when all moisture has been extracted from it. Since soil is porous its dry density is less than the density of the solid material of its constituents, which, in the majority of cases, is approximately 168 lb/ft$^3$ (2·69 g/cm$^3$). For a given dry density and a given solid density the maximum moisture content is limited by the space available in the interstices between the soil particles. On the left of the nomogram in Fig. 11 are given the limits of moisture content for materials of differing dry densities but similar solid material of density 168 lb/ft$^3$.

(4.1.2) **Kersten Formula.**

Kersten[15] developed a formula based on experimental results covering a number of samples of 19 different kinds of soil, ranging through crushed rock, pure sand ($p = 0\%$), nearly pure clay ($p = 98\%$) and peat. The measurements were for four temperatures ranging from 21° C down to −29° C, but only measurements at 21° C are taken into account as being of interest in the present work. The tests were made with samples of soil placed under controlled conditions of dry density and moisture content in a cylindrical container in which heat flow was radial. The results were analysed from the point of view of texture, size of particles, temperature, dry densities, moisture content, etc. Omitting special cases of soil with high basic igneous content and of content intermediate between basic and acid igneous rock, Kersten establishes experimentally a general formula of the form

$$k = (a \log_{10} m + b) \times 10^{\rho_d/100} \text{ milliwatts/deg C-cm},$$

where $a$ and $b$ are empirical coefficients having values as follows:

(i) For sand (granular soil of high quartz content) $a = 1·03$ and $b = 0·565$;

(*ii*) For silt and clay (having kaolin and other clay material) $a = 1·29$ and $b = -0·283$.

Kersten remarks that these equations apply for moisture content of 7% or more in silt and clay soils and of 1% or more in sandy soils. Curves covering both classes of soil are given in Fig. 8, marked KC and KS respectively.

(4.1.3) **Specific Conductivity of Soil.**

The theoretical formulae introduce the specific conductivity of the solid constituent of soil. Gemant[12,13] expresses conductivity by the formula $k_s = 58·4 - 0·33p$ where $p$ is the percentage of clay in a sand-clay mixture. Mickley[14] gives only two values: for sand $k_s = 35$ and for clay $k_s = 27·2$, from which may be deduced the law for mixtures $k_s = 35 - 0·078p$. All values are given in milliwatts per deg C-cm or thermal millimhos per cm.

(4.1.4) **Mickley's Formula.**

Mickley[14] based his calculations on a model (shown in Fig. 6) such that the thermal conductivity may be easily calculated. The

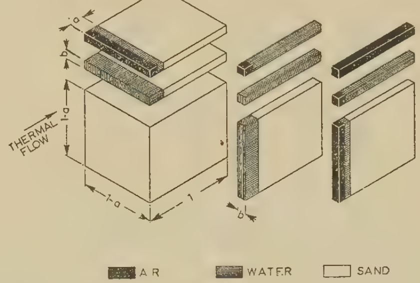


**Fig. 6.**—Elements of Mickley's unitary-cube moist-soil model.

thermal flow is along 9 parallel paths of different series combinations of solid material, water and air. As Mickley states, his formula does not hold (and over-estimates the conductivity) for very porous and very dry or nearly dry soils, since it does not take into account the fact that contacts between angular soil particles are in many cases point- and not face-contacts. As will be seen below, the value $k_s$ recommended by Mickley for sand does not seem to be the best possible choice. The curves based on Mickley's formula, for $\rho_d = 100$ lb/ft$^3$, for clay (MC), sand (MS) and for $p = 50\%$ (M 50), are given in Fig. 8. These curves are based on the thermal conductivity formula used by Gemant, as giving results in better agreement with experiment.

(4.1.5) **Gemant Formula.**

Gemant[12,13] assumes that the soil particles are of crystalline structure, with point contact between particles and that moisture collects near the points of contact in addition to being adsorbed in the form of film on the surfaces. He states that in fact nearly

all points and edges are rounded, but that this makes little difference for calculation. Because of the nature of this assumption, Gemant expects his formula to be more exact for sandy soil than for clay. The amount of water adsorbed as a thin layer on surfaces of particles depends on the temperature of the soil. Gemant equated this adsorption with adsorption of glass at the same temperature. Assuming the average size of particles usually encountered and assuming the temperature of the soil to be 10° C, it was possible to evaluate the amount of water contributing to true conductivity.

To develop his formula, Gemant analyses a unit cube (Fig. 7)

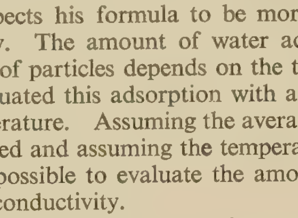


Fig. 7.—Unitary cube of moist soil according to Gemant.

composed of a cube of side $a$ of solid material, three faces of the cube being the bases of square pyramids. He divides the whole water in the soil into two parts: one adsorbed as a thin film on the surfaces on the solid particles and the second placed around the apex of each pyramid. In Fig. 7, only the water taking part in conductivity is shown. If then, unit cubes are piled one upon another and vertical heat flow is assumed, only the bottom pyramid in series with the body of the cube takes part in conduction. On this hypothesis less than one-third of the water takes part in conduction. The final formula for thermal resistivity of soil is of the form:

$$g = \frac{1}{k} = \frac{[(1-a)/a]^{4/3} \arctan [(k_s - k_w)/k_w]^{1/2}}{[m'/2]^{1/2}[k_w(k_s - k_w)]^{1/2}} + \frac{1-z_w}{k_s a} f(b^2/a \times 1) \text{ deg C-cm/watt}$$

where $a$, $b$, $Z_w$ are as shown in Fig. 7, $k_s = 58{\cdot}4 - 0{\cdot}33p$, $k_w = 6$, both in milliwatts per deg C-cm, are thermal conductivities of sand-clay mixture (with $p$% of clay) and water respectively, $f(b^2/a \times 1)$ is an experimental function of given linear dimensions $a$ and $b$, and $m'$ is the percentage moisture content after deduction of moisture adsorbed on the surface of the crystal. Examples of Gemant's curves are given in Fig. 8 for $\rho_d = 100\,\text{lb/ft}^3$ and for clay contents of 0 (GS), 50 (G50) and 100% (GC).

#### (4.1.6) Nomogram Development.

In drawing curves for thermal conductivity from Gemant's formula it was found that they may be expressed by a formula similar to Kersten's (Section 4.1.2). Thus

$$k = (a \log_{10} m + b)10^{\rho_d/100} \text{ milliwatts/deg C-cm,}$$

where the coefficients $a$ and $b$ are linear functions of clay content, namely $a = 1{\cdot}424\,08 - 0{\cdot}004\,65p$ and $b = 0{\cdot}419\,2 - 0{\cdot}003\,13p$. Being of the same form, both formulae can be conveniently represented on the same nomogram (shown in Fig. 11) where the line marked $p$ corresponds to Gemant data and the points C and S are used for Kersten's calculations for clay and sand respectively.

The additional advantage of the nomogram consists in the fact that the answer may be read either as thermal resistivity or as thermal conductivity. Dry density may be given in pounds per cubic foot or grammes per cubic centimetre. The left-hand side graph shows the limiting value of moisture content for a given dry density.

#### (4.1.7) Comparison of the Three Formulae.

Comparison of Gemant's and Mickley's proposals shows that Mickley's formula for thermal conductivity gives lower values for higher moisture content and higher values for lower moisture content. The latter difference is due to the form of the model considered by Mickley, which gives a direct path composed of solid material. The discrepancy with higher moisture content is due to the fact that Mickley uses a lower value for the thermal conductivity $k_s$ of sand. When the same values of $k_s$ are taken in both cases then, as seen from Fig. 8, the agreement

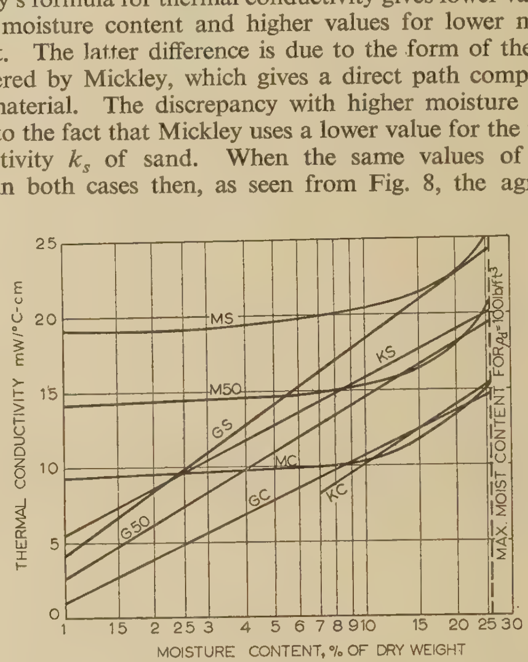


Fig. 8.—Thermal conductivity of soils having dry density $d = 100\,\text{lb/ft}^3$

$S$ = sand. $M$ = Mickley.
$C$ = clay. $G$ = Gemant.
50 = 50% of clay in sand. $K$ = Kersten.

between the two formulae is fairly good for moisture content of from 100% down to 60% of the maximum. If, however, the formulae with the same value of $k_s$ used in each are to be applied for evaluation of the conductivity of fairly dry or rather light soils only, Gemant's is suitable. This fails for dry or nearly dry soil (1–2% of moisture) however, because it does not take into account the conductivity through the path in which the adsorbed water and air are involved.

Comparison of Gemant's and Kersten's results shows that their differences are of quantitative order only. Both results can be expressed by formulae of the same kind. The dependence on dry density is exactly the same for both cases. The differences appear when the amounts of clay in sand-clay mixtures are considered. This becomes evident when the nomogram in Fig. 11 is examined. The line marked with values from $p$ = to $p = 100\%$ of clay gives data for the Gemant formula. With sand (points $S$—Kersten and $p = 0$—Gemant) both formulae give the same value of conductivity when $m = 2{\cdot}4\%$. For $m > 2{\cdot}4\%$ and $m < 2{\cdot}4\%$ Gemant's formula gives respectively higher and lower values for thermal conductivity than the Kersten formula.

For clay (point C—Kersten and $p = 100\%$—Gemant) both formulae give the same value of conductivity when $m = 14\%$. For $m > 14\%$ and $m < 14\%$ the Gemant formula gives lower and higher values respectively than the Kersten formula. Kersten limits the validity of his formula for clay to moisture contents above 7%. Indeed for $m = 2\%$ approx

* Kersten defines point C as representative of fine texture soil with above 50% silt and clay. As this assumption is indefinite the point C is compared with $p$ = 100

mately his formula predicts zero conductivity. The difference between the two formulae may be due to the fact that Kersten's division of soil groups into two categories—sandy soil and silt-clay soil—may be inadequate. Examination of soil with a large proportion of gravel (which necessarily changes the soil conductivity), the inclusion of soils of peculiar composition and the use of mean values of specific conductivity may also contribute to these differences. Since neither formula is claimed to give highly accurate results and since, as will be seen below, Gemant's formula gives results agreeing reasonably well with experiment, it appears justifiable to use Gemant's formula in practice. It is necessary, however, to remember that because of the assumptions on which Gemant's formula is based, the best results are obtained for sandy soils, relatively high moisture contents and rather high densities. The formula is probably inapplicable for moisture contents below 1 or 2% and for low dry densities.

### (4.2) Application of the Sampling Method to Field Measurements

The foregoing considerations provide a simple method of determining the thermal resistivity in the field. A soil sample is taken in such a manner that its bulky density and moisture content are preserved. The dry density $\rho_d$, moisture content $m$ and relative percentage of clay $p$ have then to be determined by analysis of the sample. The nomogram (Fig. 11) based on the Gemant formula is then used to find the value of thermal resistivity.

#### (4.2.1) Sampling Procedure.

For soil sampling the specially designed sampling tool shown in Fig. 9 has been found satisfactory. By means of this tool a

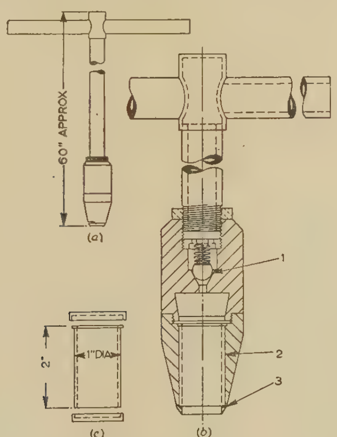


Fig. 9.—Scale drawing of earth-sampling probe.

(a) Outline of probe.
(b) Sectional elevation.
(c) Air-tight container.

(1) Ball valve and spring.
(2) Specimen container.
(3) Slot for inserting razor-blade for trimming specimen.

substantially undisturbed sample of the soil can be introduced into the airtight container fixed inside. Care must be taken to avoid large pebbles for reasons explained in Section 4.2.2. After the container has been filled, the sampling tool is withdrawn from the soil, the container is taken out, closed by lids, and stored until analysis can be made. An ordinary boring tool or spade is used to reach the depth at which the sample has to be taken. The relative percentages of clay and sand are more conveniently obtained from a larger sample taken separately by a spade or other means.

#### (4.2.2) Laboratory Analysis of Samples.

The container is cleaned and is weighed with the sample. It is then placed, with one cap removed, in a drying oven at 105–110° C and left until the weight becomes constant. If the weights before and after drying ($W_1$ and $W_2$), and the weight of the empty container $W_3$ (all in grammes) and its volume $V$ in cubic centimetres are known, the moisture content $m$ as a percentage of dry weight is found by the formula

$$m = \frac{100(W_1 - W_2)}{W_2 - W_3}$$

and the dry density by the formula

$$\rho_d = (W_2 - W_3)/V \text{ g/cm}^3$$
$$= W_2 - W_3/0{\cdot}016V \text{ lb/ft}^3$$

The analysis of the soil constitution by particle size is made generally according to the methods described for Test 8 in B.S. 1377[21] but only in as much detail as is necessary to separate the fine soil (less than 2 mm) into two groups, namely those of particle sizes respectively greater and less than 0·002 mm. Useful descriptive material is given by Wright.[19] The results of this analysis are treated in the following way. All stones and pebbles of linear dimensions greater than 2 mm are left out of consideration; it is useful to evaluate and record the amount as a percentage of the weight of air-dried fine soil (particles less than 2 mm). The particles of fine soil are then divided into two groups: sand $S$ (with dimensions between 2 mm and 0·002 mm) and clay $C$ (with dimensions less than 0·002 mm). Small amounts of organic particles, residual humidity, etc., are neglected. Then the percentage $p$ of clay in sandy soil is found from the formula $p = 100C/(S + C)$.

In general this method is satisfactory, and in the majority of cases it gives a correct answer. It must, however, be remembered that in some soils the sand (quartzite) may be crushed into small grains of a size normally attributed to clay, and conversely clay flakes (feldspar) may agglomerate into globules of the size normally classified as particles of sand. Also, silt may be of the nature of feldspar rather than quartzite. Difficulties from such diversities are likely to be infrequent, but will be avoided if, in commencing soil surveys in a given area, a few precise analyses are made, and the results used as a guide in the later work. A detailed description of soil characteristics is given in Reference 20.

A typical evaluation of clay content is shown in Table 2 for Osbaldwick. It may be noted that in spite of great variation of stone content (8–31%) the results for clay content in the fine earth are remarkably concordant. Unless stone is predominant, the error due to presence of stone should be small.

In stony soil the true dry density will probably be slightly greater than the value determined from the sample, since disturbance of the stones by the edge of the sampler may leave air pockets in the sample. On the other hand, there is a possibility that the sampler may slightly compress the sample.

Sampling errors due to compression exaggerate the dry density and are difficult to detect, but they can be avoided if care is taken not to push the tool too far into the soil. With experience, it is often possible to detect sampling errors due to voids, easily arising in stony soil, by examining the sample after drying.

## (5) TEST RESULTS

Tests have been made to ascertain that the transient-needle and soil-sampling methods are reliable and practicable for use in the field. Both methods have been successfully used in a practical investigation.

### (5.1) Practicability

Practicability of the methods has been tested by establishing a technique and making as many measurements as possible in the field, often in inclement weather and far from a base. Both methods have been found quite practicable, although neither can be satisfactorily used in rain or in waterlogged soil. Removal of the top soil with a pick is necessary in frost.

The soil sampling equipment illustrated in Fig. 9 is completely robust and, with a little practice, simple to use. A sample can be obtained from the side or bottom of an existing excavation in less than 5 min. Otherwise, the rate of sampling is largely determined by the time required to reach the site, choose the spot, and make an excavation to the required depth. Experience indicates that a responsible man and two or three labourers can obtain about four samples an hour.

The existing needles are the result of compromise between the conflicting requirements of robustness on the one hand and rapidity in obtaining reliable results which do not need correction on the other; they must not be regarded as likely to have an unlimited life. One skilled observer, one junior and one or two labourers should be able to make eight to ten observations and carry out the calculations in a day unless the soil to be measured is at considerable depth or is very stony.

### (5.2) Reliability

Reliability is best assessed by comparing results obtained by the two methods with each other and with results of sphere measurements.

Table 1 gives the results of a number of measurements with different needles and different heat inputs at different sites and at depths of 3–5 ft, all within one field and near to a cable at the C.E.A. substation at Osbaldwick, Yorkshire. The results at each site agree among themselves as well as can be expected, bearing in mind that two measurements cannot be made at the same point, that soil is not completely homogeneous, and that in this particular region the soil must have been disturbed when the cable was laid.

The results differed between sites because the soils differed, as will appear from the results of analysis of soil samples from four sites in the same field taken throughout the year 1953 (see Table 2). Table 3 presents a comparison between thermal resistivities worked out by nomogram and those measured at the same sites by the sphere and the needle methods.

Table 4 gives some results of a survey of the thermal resistivity of soil along a cable route about 1 mile long in South Durham. Soil samples were taken at 15 sites at intervals of about 100 yd and subsequently analysed in a local soil laboratory. At four points, given in the Table, readings were also taken by the transient needle for comparison.

The work occupied less than two working days. Half a day was taken by the discussion and preparation attendant on a novel project. Excavation and proving soil samples occupied two half-days of about three hours each. The needle measurements were made after the soil sampling had been completed, and occupied about three hours.

Over the period 1937–39 a number of sphere measurements on various soils were made by E.R.A. In some of these, dry density and moisture content were measured at the time and the thermal resistivity can therefore be assessed by the sampling method. Such results for two clay soils are given in Table 5. The agreement is as good as can be expected in view of the uncertainty of the soil constitution.

During the same period a number of measurements were made in sandy and somewhat stony soil at Sandy Lodge, Hertfordshire, and the results were correlated with moisture content. The results are shown in Fig. 10 for two variants of the sphere measurement. It is obvious from the dispersion of the points that dry sandy soil is a difficult subject for measurement. The correlation lines of first and second order are plotted, together with the theoretical curves of Gemant for $\rho_d = 85$, 90 and 95 lb/ft$^3$. In 1953 soil samples were taken at the same site and a few needle measurements were made. The soil samples indicated a dry density of 90 lb/ft$^3$. The agreement between the second-order correlation lines of the sphere measurements and Gemant's curve for $\rho_d = 90$ is reasonable, but not as good as it would be for a somewhat higher soil-density curve.

**Table 1**

Thermal Resistivity Measurements by Needle Method (Osbaldwick, 1953)

| Site | Needle | $I$ | Thermal resistivity |
|---|---|---|---|
| | | amp | °C-cm/W |
| 1 (September) | C | 2·0 | 62·0 |
| | C | 2·6 | 53·0 |
| | A | 2·6 | 68·8 |
| | A | 2·4 | 59 |
| | B | 3·0 | 57 |
| | | | Mean 60·0 |
| 1 (October) | G | 2·8 | 51·1 |
| | G | 2·6 | 51·6 |
| | C | 2·4 | 51·8 |
| | A | 2·4 | 51·4 |
| | | | Mean 51·5 |
| 5 | C | 2·6 | 44·4 |
| | A | 2·4 | 40·6 |
| | | | Mean 42·5 |
| 6 | A | 2·4 | 78·8 |
| | A | 2·6 | 79·8 |
| | C | 2·4 | 81·4 |
| | | | Mean 80·0 |
| 7 | A | 2·6 | 71·0 |
| | C | 2·6 | 73·0 |
| | | | Mean 72·0 |
| 8 | G | 2·6 | 38·7 |
| | G | 2·7 | 46·5 |
| | C | 2·5 | 43·8 |
| | | | Mean 43·0 |
| 10 | G | 2·6 | 58·2 |
| | C | 2·2 | 53·9 |
| | | | Mean 56·0 |

Needles A, B were provided by C.E.A. Laboratories, Leatherhead.

In the separate experiment recently made by the E.R.A. and others in which a complete heat-balance for loaded cable was determined, the value of soil resistivity necessary to satisfy the heat-balance agreed well with the value determined by needles similar to that shown in Fig. 3. This in turn agreed with the value determined by sampling, in the one instance which was confirmed.

### (6) DISCUSSION

For cable work it is generally sufficient to know the thermal resistivity of soil to within ±10%. However accurate the method of determination used, repeated tests on soils in the same vicinity are unlikely to give results more uniform than this, because of the inherent non-homogeneity of the soil. The data available

**Table 2**

SOIL PROPERTIES AT OSBALDWICK, 1953

| Site and depth | Soil composition | | | Moisture content | | | | | Dry density | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Bulk analysis | Fine earth analysis | Analysis of fine earth less residue and moisture | Feb. | April | June | Sept. | Dec. | Max. | Min. | Mean |
| | % | % | % | % | % | % | % | % | lb/ft³ | lb/ft³ | lb/ft³ |
| D<br>(3 ft) | Stones (>2 mm) 30·8<br>Fine earth (<2 mm) 69·2<br>100·0 | Coarse sand (2–0·2 mm) 28·9<br>Fine sand (0·2–0·02 mm) 34·7<br>Silt (0·02–0·002 mm) .. 10·3<br>Clay (<0·002 mm) .. 21·8<br>Moisture .. .. 2·0<br>Residue .. .. .. 2·3<br>100·0 | Sand (2–0·002 mm) 77·2<br>Clay (<0·002 mm), *p* 22·8<br>100·0 | 23·57 | 19·4 | 19·1 | 17·9 | 16·16<br>19·62 | 110·0 | 100·0 | 105·9 |
| F<br>(3 ft 6 in) | Stones (>2 mm) 8·1<br>Fine earth (<2 mm) 91·9<br>100·0 | Coarse sand (2–0·2 mm) 17·2<br>Fine sand (0·2–0·02 mm) 45·4<br>Silt (0·02–0·002 mm) .. 6·8<br>Clay (<0·002 mm) .. 26·0<br>Moisture .. .. 2·2<br>Residue .. .. .. 2·4<br>100·0 | Sand (2–0·002 mm) 72·7<br>Clay (<0·002 mm), *p* 27·3<br>100·0 | 22·75 | 20·8 | 17·35 | 17·75 | 14·94<br>12·7 | 106·0 | 90·5 | 97·1 |
| G<br>(3 ft 6 in) | Stones (>2 mm) 13·1<br>Fine earth (<2 mm) 86·9<br>100·0 | Coarse sand (2–0·2 mm) 30·5<br>Fine sand (0·2–0·02 mm) 32·2<br>Silt (0·02–0·002 mm) .. 14·3<br>Clay (<0·002 mm) .. 21·0<br>Moisture .. .. 1·7<br>Residue .. .. .. 0·3<br>100·0 | Sand (2–0·002 mm) 78·6<br>Clay (<0·002 mm), *p* 21·4<br>100·0 | 17·82 | 17·1 | 18·75<br>25·3<br>26·75 | 14·15<br>16·8<br>26·2 | 21·9<br>23·35 | 103·5 | 81·5 | 91·9 |
| S<br>(5 ft 6 in) | Stones (>2 mm) 24·4<br>Fine earth (<2 mm) 75·6<br>100·0 | Coarse sand (2–0·2 mm) 33·0<br>Fine sand (0·2–0·02 mm) 29·0<br>Silt (0·02–0·002 mm) .. 12·3<br>Clay (<0·002 mm) .. 22·5<br>Moisture .. .. 1·9<br>Residue .. .. .. 1·3<br>100·0 | Sand (2–0·002 mm) 76·8<br>Clay (<0·002 mm), *p* 23·2<br>100·0 | 35·64 | 23·6 | 14·4 | 24·2 | 21·4<br>18·94 | 108·0 | 90·5 | 96·9 |

Average value of $p$ in column 4 = 23·7%.

## Table 3

Thermal Resistivity of Soil at Osbaldwick in 1953, obtained by Different Methods

| Site | Month | Thermal resistivities | | | |
|---|---|---|---|---|---|
| | | Sampling method | Sphere method | Cylinder method | Needle method |
| | | °C-cm/W | °C-cm/W | °C-cm/W | °C-cm/W |
| G | Feb. | 60 | 57 | 62 | — |
| | Apr. | 61 | 56 | 62 | — |
| | June | 54 | 64 | 77 | 55 |
| | Sept. | 60 | 170* | 110* | 60 |
| | Dec. | 57 | 55 | 65 | — |
| F | Feb. | 49 | 54 | — | — |
| | Apr. | 58 | 54 | — | — |
| | June | 53 | 57 | — | — |
| | Sept. | 54 | 140* | — | 80 |
| | Dec. | 58 | 152* | — | — |
| S | Feb. | 45 | 55 | — | — |
| | Apr. | 50 | 53 | — | — |
| | June | 57 | 69 | — | — |
| | Sept. | 49 | 110* | — | — |
| | Dec. | 52 | 50 | — | — |
| D | Feb. | 41 | 49 | — | — |
| | Apr. | 43 | 51 | — | — |
| | June | 42 | 47 | — | — |
| | Sept. | 44 | 50 | — | 52 |
| | Dec. | 44 | 50 | — | — |

* Moisture runaway conditions.

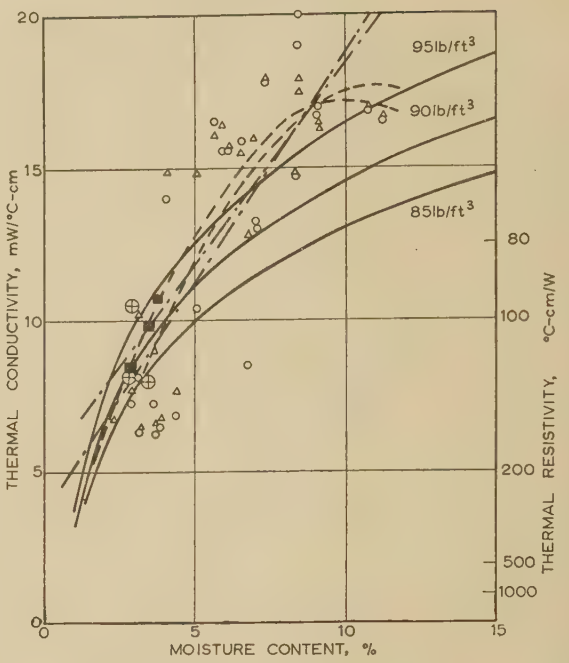


**Fig. 10.**—Comparison of observed and calculated values of therma conductivity of pure sand based on results obtained at Sand Lodge, Hertfordshire.

○ 9 in sphere test; temperature measured at surface of sphere.
△ 9 in sphere test; temperature measured 6 in from centre of sphere.
■ Points by sampling method.
⊕ Points by needle method.
——— Gemant's curves.
—·—· 1st-order correlation line.
----- 2nd-order correlation line.

## Table 4

Measurement of Thermal Resistivity at Spennymoor by Needle and Sampling Methods

| Site | Dry density $\rho_d$ | Clay content, $p$ | Moisture content, $m$ | Thermal resistivity | |
|---|---|---|---|---|---|
| | | | | By sampling method | By needle method |
| | lb/ft³ | % | % | °C-cm/W | °C-cm/W |
| 4 | 85·5, 92·2 (mean 88·85) | 40 | 25·56, 28·0 (mean 26·78) | 63·5 | 57 |
| 8 | 88·6, 84·4 (mean 88·5) | 22 | 26·8, 27·01 (mean 26·9) | 58 | 65, 58 |
| 11 | 74·6 | 49 | 33·45 | 84·5 | 84 |
| 12 | 78·2, 73·1 (mean 75·65) | 42 | 30·37, 31·18 (mean 30·75) | 82 | 69 |

Average percentage of pebbles (>2 mm) is 2·5%.

show that both the methods discussed in the paper give results broadly agreeing with those obtained by the sphere and with each other within these limits, except in extreme conditions.

For a cable route a reading at a single site will rarely be sufficient. When a number of measurements are made on several sites any apparently anomalous readings should be reinvestigated before the survey is considered closed. Hence the necessity for apparatus giving immediate answers without lengthy calculations.

The slightly greater consistency obtained with the needle method can be explained by the fact that a greater bulk of the soil is subject to test and this tends to smooth out local non-uniformities. To the extent that the theory of the needle meth may be regarded as flawless and that the instrument and t measuring circuits represent the highest attainable skill, th method may be considered to rank as a standard even to t exclusion of the sphere method. With the sphere method, d turbance of the ground when laying, and local soil-drying af prolonged loading may be considered to be disadvantages.

With the sampling method bad sampling causes errors in t measured dry density and consequent errors in calculating t thermal resistivity. The measurement of moisture content, ho ever, is free from such errors. There is therefore a possibility using the sampling and needle methods in conjunction, referr

Table 5

SOIL CHARACTERISTICS AND THERMAL RESISTIVITY OF LONDON AND BEDFORD CLAY

| | London clay (late 1937) | | Bedford clay (November, 1939) | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 1 | 2 | 3 | 4 |
| Dry density g/cm³ .. .. .. | 1·33 | 1·2 | 1·53 | 1·53 | 1·60 | 1·45 |
| Dry density lb/ft³ .. .. .. | 83 | 75 | 95·5 | 95·5 | 100 | 91 |
| Moisture content (% of dry weight) .. | 37·7 37·2 | 45·1 | 30·6 | 30·7 | 22·6 | 33·2 |
| Thermal resistivity °C-cm/W calculated from the nomogram (a)* | 92 | 105 | 74 | 74 | 71 | 80 |
| Thermal resistivity °C-cm/W calculated from the nomogram (b)* | 73 | 84·5 | 58 | 58 | 56 | 62 |
| Thermal resistivity by sphere °C-cm/W | 81·3 | | 70 | | | |

* No soil analysis was made at the time of these tests.
The amount of clay assumed in the soil is as follows:
(a) $p = 100\%$.
(b) $p = 60\%$ (London). } Average figures given by D.S.I.R.
$p = 56\%$ (Bedford). } Road Research Laboratory

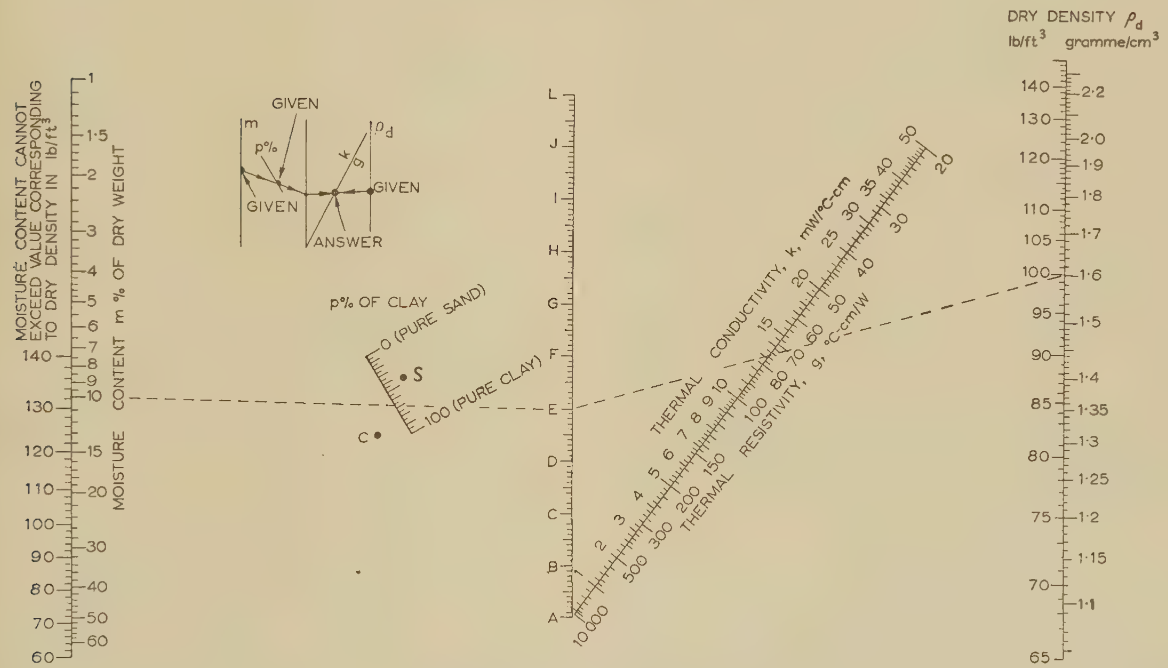


Fig. 11.—Nomogram for determination of thermal conductivity and resistivity of sandy soil, given the dry density, moisture content and percentage of clay.

Example: If soil has
65% clay,
10% moisture content,
and 100 lb/ft³ = 1·6 g/cm³ dry density,
$k = 13 \cdot 1$ mW/°C-cm; $g = 76 \cdot 5$ °C-cm/W.

The given moisture content $m$ and dry density $\rho_d$ must be checked for consistency by graph on left-hand side; e.g. if dry density is 132 lb/ft³ moisture content cannot exceed 10%.

one particular reading of thermal resistivity to the moisture content and accurate dry density taken at the time of the reading and subsequently obtaining thermal resistivities by correcting for the moisture content of the soil.

Neither of the methods described takes into account the effects of moisture movement away from the heated body. This may be of importance in fairly dry soils. Moreover, the theory of the sampling method leads to rather inaccurate estimates of the effective resistivity of very dry soils. The sphere method can register the migration of moisture, but from geometric considerations moisture migration is likely to have different effects upon the resistivity for a sphere and for a long cylinder, such as cable, at the same surface temperature.

For cables the measurement should, if possible, be made at the time of year corresponding to that at which the installation in question is likely to be most heavily loaded, or in early autumn when the ground is usually in the driest condition. For pipes conveying hot liquid or steam, or drawing heat from the soil, most unfavourable conditions would probably occur some time during winter.

The costs of the two methods may vary considerably according to the local conditions, but they are generally of the same order.[3]

## (7) CONCLUSIONS

From examination of the two methods—the transient-needle method and the soil-sampling method—of rapidly assessing thermal resistivity of soils, with particular application to the determination of thermal ratings of buried cables, the following conclusions may be drawn.

(*a*) Both methods can give accuracy and repeatability up to the standard at present thought sufficient, namely about $\pm 10\%$ of the true value.

(*b*) The needle method is applicable to any soil into which a probe can be inserted to make good contact, and gives an immediate determination of resistivity from first principles quickly and with good accuracy. It requires rather precise measurements under field conditions, and gives none of the information about the nature of the soil usually necessary for assessing the probable seasonal variations.

(*c*) The soil-sampling method is so far applicable only to soils of sand, clay and mixtures of sand and clay. It has the advantage that samples can be taken from a trench or from the surface by means of a robust tool and that measurements are not made in the field. Although the thermal resistivity cannot be made known in less than 7–10 days the method permits an estimate of seasonal variations in thermal resistivity. Whilst skill is required in choosing sites, in intelligent sampling and in the laboratory analysis of soil, organizations accustomed to such analysis probably exist in every county, and basic data on probable moisture content and soil constitution may also be obtained from other authorities such as road engineers.

(*d*) The two methods may be used successfully in conjunction by referring one particular thermal resistivity measurement taken by the needle method to a moisture-content and an accurate dry-density measurement taken on a sample obtained at the same time. Subsequently, by measurement of the moisture content in samples taken from the same spot, thermal resistivity can be determined at any time.

(*e*) Neither method is directly practicable in a soil containing so much stone or chippings that a needle or a sampling tool cannot be inserted without gross disturbance of the soil.

(*f*) Neither method takes account of the movement of moisture away from heated bodies over a period of time.

## (8) ACKNOWLEDGMENTS

The authors are greatly indebted to Mr. L. Gosland, under whose direction the work described was carried out, for invaluable help and for constructive guidance in the preparation of the paper.

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# DISCUSSION BEFORE A JOINT MEETING OF THE MEASUREMENT AND CONTROL SECTION AND THE SUPPLY SECTION, 28TH FEBRUARY, 1956

**Mr. W. H. Lythgoe:** The paper is very opportune, as one of several means for obtaining more economic use of the copper which is installed and any which subsequently has to be put into the ground.

The question of soil thermal resistivity is not new. E.R.A. reports on the subject of current rating go back for a number of years. They were first presented in 1921 and 1923, when reference was made to a plate system which had been developed at the N.P.L. The variations in the soil and its moisture content and indeed the variations of the materials in the soil, were

discussed, but everyone was a little uncertain. People were relieved to have some authority which stated what cable ratings could be, and they rather hesitated to probe into this rather complex problem.

In 1938 a further E.R.A. report appeared in which the sphere method was advocated, and in the development of which Dr. Griffiths played a very prominent part. At that time it was left very largely to the cable industry to conduct any measurements of soil thermal resistivity which were required. It is unfortunate that, although a large number of measurements were taken over the years, too much was left to the individuals concerned without adequate guidance. Certainly the records of those tests bear very slight examination, because too many differing phrases without adequate classification were used to describe the types of soils. In addition, of course, insufficient attention was paid at that time and for many years after to the variations in soil moisture content. Thus a great deal of the work which was carried out is of very doubtful value, except to give an indication of the variation in soil thermal resistivity which is found and the preponderance of certain values, particularly in the region of $g = 80$–$120°$ C-cm/watt. Those tests have shown extremely variable results. I recollect values lower than 60, and as high as 300 plus. Because of the published values, based on $g = 120$, many people appear to have assumed that this was a standard soil thermal resistivity from which no departures were ever likely to be made.

More recently the problem has become increasingly the concern of the supply industry itself, because of the greater extent of their own installations.

I think that the authors and the E.R.A. would be the last to suggest that this is a final report on this very vexed subject; it should be regarded as an interim report. I am convinced that there is a field for the time being for all three methods, i.e. the sphere, the needle probe, and the sampling method. Therefore, I would plead that no particular selection be made of any one method as against another, because of cost, convenience or prejudice, but that an honest attempt be made to assess the merits of all three.

The difficulty with the sphere method is that insertion of the sphere produces a major disturbance of the ground. When prospecting a new site or route one frequently has to take rather careful and expensive precautions in order to maintain the sphere and the associated equipment in position for a period of about a week, during which time measurements are taken.

On the other hand, the needle method does give a chance of making a rapid assessment, but without definite knowledge of the soil structure. It must also be admitted that there are large numbers of soils for which the probe is not suitable because it is a relatively flimsy device. I hope that, in due course, means will be found to strengthen the needle, which is a very valuable tool, and to make it more robust. The sampling method, again, has its limitations where the nature of the soil is particularly stony. I recollect some sites examined in the past which are of a distinctly rocky nature for which only the sphere method would be feasible.

**Mr. C. C. Barnes:** It is important to remember that measurements have no virtue in themselves; it is their utilization that is of real importance. It is desirable, therefore, that engineers closely concerned with measurement techniques should take a keen interest in the end-product of their activities. My observations are concerned with the optimum utilization of the current loading of power cables.

According to the Monopolies Report on the Supply of Insulated Electric Wires and Cables, the total value of the supply of mains cables in the United Kingdom in 1948 was in excess of £17 million, and a large proportion of this cable must have been buried direct in the ground. Therefore, any means of increasing the permissible current rating of buried cables will result in a large financial saving and a greater utilization of the cable materials.

A further factor of continuing importance, at present, is the high price of copper, lead and steel, which are the major components of power cables.

Fig. A shows the variations in metal prices over a period of 40 years. The present figure for copper is about £440 a ton, and for lead it is nearly £120 a ton. This rapidly increasing price trend has caused attention to be focused on the need for looking very carefully at the way in which the copper in the cable is used.

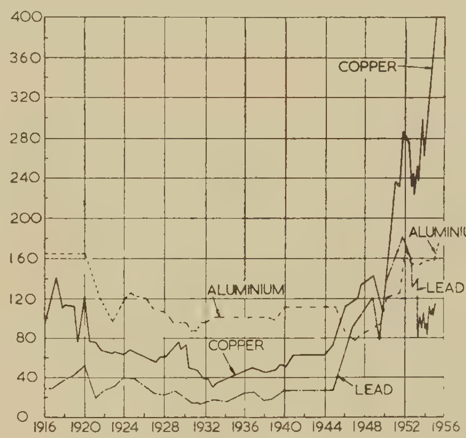


**Fig. A.**—Curves showing the general trend of metal prices. Average prices of copper, soft lead (ingots) and aluminium 1916–1955.

The rapid increase in metal prices in September, 1949, followed the devaluation of the pound sterling.

A fundamental factor affecting the permissible current loading is the soil thermal resistivity, and the standard value in Great Britain for many years has been 120° C-cm/watt. This value was determined on the basis of investigations carried out by the E.R.A. and the C.M.A. over many years, but a limited number of tests taken in the last two or three years by the C.E.A. and Area Boards have shown that a value of $g = 90°$ C-cm/watt can be safely used in many cases.

For soils having a value of $g$ differing from the normal basis of 120° C-cm/watt, current-rating factors are approximately 1·10 and 1·05 for soil thermal resistivities of 90 and 100° C-cm/watt, respectively. The magnitude of the financial saving will be readily appreciated.

This issue has therefore been actively stressed by cable users in the appropriate E.R.A. Technical Committees, and in May, 1955, E.R.A. Report F/T183 was issued which contains two tabulated sets of current loadings for cables laid direct in the ground, or pulled into underground ducts, based on two alternative values of soil resistivity, namely $g = 120$ and 90° C-cm/watt, respectively.

It is of interest to note the standard soil thermal resistivities used by other countries. They are as follows:

| | |
|---|---|
| France .. .. | 85° C-cm/watt |
| United States .. | 80–90° C-cm/watt |
| Sweden .. .. | 100° C-cm/watt |
| Italy .. .. .. | 100° C-cm/watt |

In the German V.D.E. Specification 0255/1951 a value of $g = 70°$ C-cm/watt is used.

In order to ensure that full advantage is taken of the permissible

current-carrying capacity of buried cables for all new installations, soil thermal-resistivity measurements should be an essential preliminary requirement. It follows, therefore, that the paper justifies careful and critical examination.

For many years the standard method of measuring soil thermal resistivity has been by the sphere or cylinder technique, which requires five to seven days in order to obtain results. Essential requirements for field tests are speed, simplicity and reproducibility in the testing methods, together with mechanical robustness in the construction of the equipment used. The need to conserve testing time and the cost of soil-thermal-resistivity tests have resulted in the development of quicker and less expensive methods.

Nevertheless, in view of the many results already available based on sphere measurements, it is desirable to have detailed information showing how these results compare with tests taken under identical conditions with the transient needle and/or soil sampling techniques—the data given in Tables 3 and 4 are interesting but far too limited to provide definite conclusions, and I am interested to know whether the authors have now obtained more comparative test results.

In Section 1 the authors refer to the need for making allowance for moisture movement in the case of heavily loaded cables. I would like their suggestions for dealing with this vital problem.

The measured soil thermal resistivity will vary at different times of the year, mainly owing to the moisture content of the earth. I would, therefore, welcome from the authors test data showing comparative soil thermal resistivities obtained throughout the year, in order to get an indication of the variation which may be expected for different types of soil. Fig. 13 of E.R.A. Report F/T128 contains limited information on the issue, but this Figure was based on test results taken in pre-war days, and modern test data are desirable.

The authors' observations in Section 5.1 with regard to the short time required for obtaining test results with the needle technique should greatly encourage cable users to make increasing use of such testing methods, with the object of achieving more efficient use of the copper provided in buried cables.

**Mr. F. R. Axworthy:** The transient-needle method of determining thermal resistivity, developed by the authors, provides a rapid and convenient method of measurement, and on reading the draft E.R.A. Report on the subject, it was clear that the convenience of the method would be enhanced if the various components were made into a complete test set. For such a test set, a direct-reading temperature indicator is to be preferred to the potentiometer measurement required by the authors' thermopiles. Such a direct-reading instrument is best achieved by a resistance bridge, and with a good deal of help and advice from Mr. Mochlinski, I produced some resistance-bridge needles, based on his design.

It was therefore possible to produce a complete test set for making these measurements, comprising the heater circuits with ammeter and current controls, a direct-reading resistance bridge and resistance-thermometer needles. There seems little doubt that such equipment is extremely convenient for field testing, and seems to be of sufficient accuracy. The actual accuracy achieved seems to be nearer $\pm 15\%$ than $\pm 10\%$, but I suggest that, in view of the inherent uncertainty of measurements of this nature, $\pm 15\%$ is sufficiently accurate for the purpose.

**Dr. E. Griffiths:** I am interested in the assessment of thermal resistivity from the constitution of the soil.

With regard to the transient needle-method, in any resistivity measurement it is advisable that the dimensions of the heat source should be large compared with the grain size of the material under test. I suggest that the probe of $\frac{3}{16}$in diameter is suitable for fine-grain material only.

I was responsible for the introduction of the sphere method referred to in the paper, and I worked out the details in the early days. I favoured the use of a sizeable hot object—in this case spheres of 6 or 9in diameter.

Further work under laboratory conditions would supply much useful information which could lead to a reduction in the number of field tests.

The effect of density of packing of the soil should be studied. On one occasion we studied a soil of various densities, which was obtained by loading the flat test plates up to about 1 ton/ft$^2$.

The authors stress the fact that the transient-needle method does not involve disturbance of the moisture distribution. When a cable has been in operation for a time there is moisture movement, and these are the conditions that should be simulated, if possible. With the sphere method one can detect whether there is moisture movement.

Thermal-diffusivity determinations have been mentioned. I adapted the sphere method for these; the surface of the sphere was so heated that its temperature followed a sinusoidal curve. The measurements provided the data for calculating diffusivity.

**Mr. F. C. Hole:** The London Electricity Board has been trying out the practical application of the probe method with probes of the size shown in Fig. 3 and having various thermocouple arrangements. The probe with the Fig. 3 construction gives comparative results about 6% high.

Tests on probe design were carried out in four artificial soils, having the following approximate thermal resistivities in °C-cm/watt (as measured by the probe):

| | |
|---|---|
| Dry silver sand .. .. .. | 275 |
| Oil-saturated cement powder .. | 175 |
| Oil-saturated silver sand .. .. | 75 |
| Water-saturated silver sand .. | 37 |

Robustness is improved by filling with synthetic resin, which prevents moisture troubles, and by shortening the length for the same diameter with the possibility of reduced accuracy. Both 12in and 18in probes with and without this filling give results to within 10% of those of the type F design in E.R.A. Report F/T181.

The field-testing technique includes initially loading the accumulator at the rate required for the probe and avoiding purely local effects by using probes permanently connected in pairs, each in turn functioning as the reference junction. In a field test the original probes gave results some 30% lower than that of a 3in sphere with a moderate surface temperature. As one of the two comparative results given in Table 3 of the paper also shows a low probe result, I should like to have the authors' observations on this point.

With reference to Section 3.3.3, I should be interested to know the likely variation in temperature between the two sides of the probe. The output of a single thermocouple can be measured satisfactorily, and the omission of one thermocouple simplifies construction and eliminates one weak spot in the wall of the probe. Temperature measurement by means of a resistance-bridge method may be preferable for field use.

With reference to Section 4.1.1, could the authors enlarge upon their statement about the effect of pebbles when their proportion is increased to the stage when there is an inadequate amount of soil to fill the interstices?

**Dr. K. Konstantinowsky:** Permissible continuous loading of a buried cable is controlled by the sum of two terms—the internal and external thermal resistances of the cable. The internal thermal resistance is fairly well known and settled, and the external thermal resistance is clear so far as the structure of the formula is concerned, but the soil thermal resistivity is very much in doubt. Soil thermal resistivities quoted vary between 70 and 180°C-cm/watt.

The two methods described for determining the soil thermal resistivity, $g$, are new in this country, and if used separately are probably not too safe. I believe the answer will be found in a combination or simultaneous application of both methods.

The sampling method, applicable only to certain soils, needs more work to make wider application possible. It makes use of an empirical formula, and Fig. 10 shows results from empirical formulae given on certain assumptions. Actual test results do not differ widely up to, say 8%. Beyond this region $g$ is low and not to such an extent influencing the permissible loading of the cable.

The authors rightly state that the needle method, using the transient part of the heating process, gives quicker and more reliable results. Of 16 measurements only four are carried out by the needle method (see Table 3). This is unlikely to induce confidence in the method. Of these four results two are not comparable with the result given by the sphere method because of 'moisture runaway' conditions. Could the authors explain more fully the 'moisture runaway' conditions? Further tests given in Table 1 appear fairly good and consistent, but again only one can be identified in Table 3.

A previous speaker advocated using a resistance-temperature measurement instead of thermocouples—a method used by us in 1944 for a similar purpose. The transient part of the heating was used, with two expedients which in my experience are not often used. The same wire was used for heating and temperature-resistance measurement but with a 'false zero point' bridge. Heating and cooling curves were taken and extrapolated to switching-off time. From the two temperature/time gradient curves we obtained the thermal resistivity of an insulated wire. Results showed that correct values were obtained after a few seconds. By other methods, thermal resistivities found for a particular p.v.c. mixture were 600°C-cm/watt for a wire insulation and 670°C-cm/watt for sheet material, while the figure found by the quick method was 620°C-cm/watt for wire insulation.

**Dr. F. Busemann:** With regard to Mr. Lythgoe's remarks on cyclic rating, we need to know not only the thermal resistivity of the soil but also the diffusivity $D = 1/(gc_v)$, where $c_v$ is the specific heat per unit of volume. The specific heat of the solid matter of soils seems to vary very little around a value of 0·2cal/g°C. This, together with the specific heat of water which is unity, gives a simple formula

$$c_v = (1 \cdot 34 + 0 \cdot 67m/10)\rho_d/100$$

For an average soil of density 100 lb/ft$^3$ and 10% moisture, $c_v \simeq 2$ watt-sec/°C-cm$^3$. For the practical range of soil it varies from 1·5 to 3 watt-sec/°C-cm$^3$. I should recommend a value of $c_v = 2$ watt-sec/°C-cm$^3$ as a rough estimate, which makes the diffusivity $D = 1/(2g)$ cm$^2$/sec.

In Section 3.2 the authors state that the ratio of length to diameter should be about 100 : 1. What are the reasons for this, and did the authors obtain it from theoretical considerations or practical tests? The needles quoted are all made of materials like brass and Duralumin which have a fairly high conductivity. Perhaps if another material, such as steel, were used we could reduce the end effect and obtain a shorter and a more robust needle.

The theoretical curves of Fig. 1 start horizontally and turn upwards, but the practical curves of Fig. 2 start by going upwards and then flatten out. This difference between the practical and theoretical curve is due to an extra temperature rise, probably owing to the arrangement of the thermocouple in the needle. The thermocouple thus measures a temperature higher than that used in the calculations. We base the final value of the thermal resistivity on the measurement of a tiny difference from a large temperature rise. I estimate that for curve ($a$) it is about 2° out of 12°, and it requires extra good accuracy of measurement and test conditions. I wonder whether the authors think that there is some scope for improvement.

Have the authors any suggestions for hard and stony soil?

**Mr. H. N. Cox:** It is not unusual for further cables to be required along a route on which cables are already in service. In such a case fairly precise knowledge of soil thermal resistivity would be particularly valuable. It should be recognized that the natural resistivity may have been modified by the effects of heat from the existing cables, and that the needle and sampling methods of measurement might not be directly applicable in soil in which there is a temperature gradient due to loaded cables.

Possibly the transient-needle equipment would still prove reasonably accurate in such circumstances, and the soil sampling method might be applied with nomograms modified for temperatures greater than the 10°C presupposed in Fig. 11. Perhaps the authors would give their views and also state whether, in the Osbaldwick measurements (Section 5.2), there were any heating effects from the adjacent cable.

The following example may illustrate the importance of soil thermal resistivity on congested routes. On the basis of the conventional assumption that $g = 120$, a current of 460 amp can be carried by a thermally independent 33 kV 3-core 0·5 in$^2$ solid-type cable. However, if loads of 460 amp were to be carried by each of four circuits, at 18 in centres, then, on account of mutual heating, it would be necessary to use single-core 1·0 in$^2$ cables. If $g = 60$, 0·5 in$^2$ 3-core cables would be adequate.

Another example may indicate the need for caution in taking account of low values of $g$ on single-circuit routes. With $g = 120$, 0·5 in$^2$ 33 kV cable would be required for a 460 amp load; with $g = 90$, 0·4 in$^2$ cable could be used, giving a price saving of about £1 000 per 1 000 yd, but this would be counterbalanced by the cost of the 11 kW per 1 000 yd additional full-load losses.

**Mr. A. H. Bennett:** We want as many people as possible to carry out tests. In the old days many of the undertakings were run by municipal corporations. They had a chief engineer and a mains engineer and it was usual to pass as much of the responsibility as possible over to the C.M.A. or the cable manufacturer. There is now only one electricity supply authority in Great Britain, and its Divisions and Area Boards have a very large number of mains engineers. I appeal to them to make use of the tests which have been described in the paper, and let the E.R.A. have them on the form which has already been prepared.

We are too worried about the overloading of cables. At the beginning of the century, cables which were laid for 11 kV working were lead covered and laid in stoneware troughs or sometimes in a concrete trough. They were supported in these troughs, and approximately $\frac{1}{2}$–$\frac{3}{4}$ in of heat-insulating compound was poured round them. There was not only the difficulty of getting the heat away through the insulation of the cable but there was also the pitch and anything up to a little over $\frac{1}{2}$ in of stoneware. I have known 11 kV cables to carry 250 amp on 0·1 in$^2$ for hours and hours during the First World War, and they are still working.

**Dr. M. S. Kersten** (*Minnesota, United States: communicated*): I should like to emphasize the importance of Section 7($c$). We have, in effect, used this method in many studies in Minnesota. In some instances we base our selection of $k$ on moisture-content samples plus a manual determination of texture plus an estimate of density based on experience. We do not know if we attain values ±10% of the true value. We are probably in more error at times, but in view of the variations in the soil patterns encountered we feel they are still adequate.

One is occasionally troubled by some apparent discrepancies

between estimated and measured conductivities, as indicated in Fig. 10 for the sands with moisture contents between 5 and 10%. In this particular instance, and admittedly basing an opinion on a very limited description, I would first be inclined to check further on the density. Only a single value is given—90lb/ft³—for sands at a wide range of moisture contents, and further density testing might possibly indicate some higher values. If densities of 100 or 110lb/ft³ were found, the check between estimated and observed conductivities would be very good. Certain sands with high quartz contents might also give high conductivities.

The Minnesota $k$-values (see Reference 14 or 15 for charts of values) are based on tests of 19 soils at a great variety of moisture content and density conditions; over a thousand determinations of $k$ were made. It is remarkable to note the check between values from these charts, which are essentially empirical in nature, and the values given by Gemant's formula based on the unitary cube. It should be remembered that some variation in results may be obtained on soils of the same texture and at equal moisture contents and densities; this may be due to mineral composition, particle shape, etc. Hence too many refinements should not be introduced into charts, since values deduced from them must be considered as 'average' for the measured conditions.

**Mr. L. H. Fink** (*Philadelphia, United States: communicated*): The authors outline several convenient advances in techniques for evaluating soil resistivity over those summarized by Mickley.* The thermal needle is unique in regard to certain of its advantages. Gemant's soil model is certainly a much closer approximation to the structure of soils than Mickley's, and the authors' reduction of the corresponding formula to nomographic form provides a valuable and convenient tool for estimating the thermal resistivity of a given soil.

The scope of the paper, however, would appear to be somewhat more restricted than that of Mickley's. The latter's procedure, as summarized in his Table VI, provided for a 4-way evaluation of soils. The considerations involved were the dry density of the soil, the probable minimum moisture content, the estimated thermal resistivity, and the estimated effect of moisture migration. In the present paper, as stated in Section 7($f$), no account is taken of moisture migration, and in lieu of the probable minimum moisture content, use is made of the moisture content obtaining in the field at the time the sample is taken.

We are at present in little better position to estimate the effect of moisture migration than when Mickley's paper was presented, and the empirical factor $K_m$, to which he had recourse, is still perhaps the best available tool if moisture migration is to be considered.

Dependence on the actual moisture content of the field sample as taken would appear to be a questionable practice. As pointed out in the paper, the moisture content of soils in the field can vary with time over a wide range. Cable engineers, however, are concerned primarily with the minimum moisture content which can be expected under normal field conditions, i.e. what the moisture content of the soil is likely to be during the driest part of the year. Under the procedure recommended by the authors, this could only be estimated by repeated measurements of moisture content over a long period of time. Obviously such a procedure would not provide the rapid evaluation which is the desideratum of the paper. It would seem that the procedure recommended by Mickley, involving the use of the tension table developed by Leamer and Shaw,† still provides the most convenient approach.

The thermal needle provides the most convenient available means for making a rapid evaluation of thermal resistivity over an extended area. However, as pointed out by the authors, extreme care in its use is required, with regard both to the physical placement of the needle in the soil and the measurement of the heater current, if it is to provide even moderately reliable data. In addition, the instrument provides no indication of either the density of the soil or its moisture content. If the needle is to be used for anything other than the sort of qualitative preliminary survey for which it is best adapted, density and moisture must be determined by standard techniques, and the advantage of the needle is lost. This disadvantage may be overcome with further improvement and use of the instruments now beginning to appear, which utilize the back scattering of neutrons and $\gamma$-rays from a suitable radioactive source in order to provide a direct indication of soil density and moisture content.

There is one further consideration which has not been treated in either the present paper or that of Mickley. Both the methods discussed in the present paper, as well as those recommended by Mickley, are based on small samples which are intended to represent the resistivity of soil over a large area. Whenever conclusions must be drawn from small samples, a careful consideration of the principles of statistical inference will indicate how the most information can be obtained and what confidence limits may be attached to that information. To my knowledge, no such study has been made with respect to the sampling of soil along the route of a proposed cable line when an estimate of the overall variation in soil characteristics is desired.

**Dr. A. Gemant** (*Michigan, United States: communicated*): When I developed the theory upon which the present paper is partly based, my main purpose was to fill an apparent gap in the basic knowledge of soil thermal conductivity. This was done by developing a formula yielding calculated conductivities that, within certain limitations, agreed satisfactorily with experimental data of the literature.

It was realized at that time that the formula as developed could be used for the determination of soil conductivities from the characteristics of the soil.* However, no further steps were taken in that direction.

It is gratifying, therefore, to note that the authors went to the trouble of developing a simple procedure of obtaining the soil characteristics, and deriving a nomogram from the above-mentioned formula which allows the determination of soil conductivities without further calculations. I also agree with the authors that such a technique might be found particularly valuable in cable-loading operations.

**Prof. F. C. Hooper** (*Toronto, Canada: communicated*): There has been a real need for a comparative study of the probe and sampling techniques for thermal-conductivity determination in soils, and the authors' work has to a large degree filled that need. Those of us who have been active in the development of the probe technique and have recommended its use for several years are pleased to find support in the conclusions of the paper.

I believe that the question of moisture migration deserves further study. The authors state that neither method takes into account the movement of moisture away from heated bodies over a period of time. This is true if it is taken to mean that they do not yield a single coefficient which, applied in an equation which assumes homogeneous moisture distribution, will give the proper result in a non-homogeneous and probably transient condition. This would appear to be an impossible demand.

In a moist soil it must be recognized that, under many conditions, heat is transferred not only by conduction but also by an evaporation-mass-transfer-condensation mechanism. The probe result will include at least a part of the heat transfer due to the latter mechanism, and by virtue of its short time of

* Mickley, A. S.: 'The Thermal Conductivity of Moist Soil', *Transactions of the American I.E.E.*, 1951, 70, p. 1789.

† Leamer, R. W., and Shaw, B.: 'A Simple Apparatus for Measuring Noncapillary Porosity on an Extensive Scale', *Journal of the American Society of Agronomy*, 1941, 33, p. 1003.

* Gemant, A.: *Heating, Piping and Air Conditioning*, 1952, 24, p. 122.

operation will avoid the establishment of a non-homogeneous moisture field in its own vicinity. Properly used in conjunction with observed moisture distributions in the region of a heat source, the probe results could be used to predict the effect of the moisture-migration phenomena.

**Dr. J. H. Blackwell** (*Canberra, Australia: communicated*): All the comments refer to the transient-needle or thermal-conductivity-probe method.

In Section 3.1 the authors are, of course, fully aware of the extreme idealization involved in using line-source theory. Therefore why do they use it? The first correction term to the simple logarithmic relationship, at large times, involves the probe parameters and is quite different in form from the first correction term in line-source theory. This first correction term cannot be ignored if the ultimate accuracy possible with a given set of experimental data is required. At the University of Western Ontario we consider it very difficult indeed to draw the best straight line by eye through the large time points. One has to consider not only the experimental scatter of the data but also the increasing departure of the points from the theoretical straight line, as time decreases. We have taken sets of our own data and others from published papers, and two of us have independently attempted to determine the slope by this method. Variation of 3% or 4% between ourselves, and with the published values, have been common. (It is admitted, though, that the scatter in the points used during these tests seemed, in general, to be greater than that in Fig. 2 of the paper.)

At the cost of a little extra computing time, this ambiguity can be almost entirely removed by fitting the first and second terms of the large-time relation. In effect, this process about doubles the number of experimental points available for the determination of conductivities.

With reference to Section 3.2, the relative axial-flow error in a probe naturally is not a function of the ratio of length to diameter alone. Recent work* has shown, however, that previous estimates of the required value of this ratio have tended to be conservative. As an example, a $\frac{1}{4}$ in solid brass needle for use in soils ($k \simeq 0\cdot002$ C.G.S. unit, $D \simeq 0\cdot005$ C.G.S. unit) need not be longer than $7\frac{1}{2}$ in, for an axial-flow error in the slope of less than $\frac{1}{2}$% at $t = 500$ sec. For the error to be less than $\frac{1}{2}$% at $t = 2\,000$ sec, the probe should be 14 in long, but even here the ratio of length to diameter is only 56; this value of $t$ incidentally corresponds to a $Dt/b^2 = 100$, which is far larger than is normally used in an experiment. Since the needles are hollow rather than solid, the requirement on length is still further reduced.

Although, for work in soils, the boundary contact-resistance is usually quite low and may probably be ignored so far as conductivity determination is concerned, it appears that it will have to be considered if diffusivity determination is required. I suspect that neglect of this phenomenon is one basic reason for the present scepticism towards diffusivity results obtained with the transient needle. I am at present attempting an investigation of the accuracy of diffusivity measurement.

With regard to Section 3.3.3, I am strongly of the opinion that, for field work at least, a thermistor resistance thermometer has several advantages over the use of thermocouples.

If measurement to $0\cdot1°$ C only is required, the bridge necessary can be contained in a case no bigger than that of a standard portable meter, and by displaying the off-balance current on a microammeter the measurement procedure is reduced simply to reading the meter as a function of time. (The speed of this process is particularly useful when measuring the rapidly changing temperatures at small times, as is required if a contact-resistance determination is to be made.)

If measurement to $0\cdot01°$ C is necessary, the simple d.c. bridge must be replaced by an a.c. bridge; even in this case, if miniaturization is properly carried out (e.g. replacing valves with transistors) it should be possible to keep the bulk and weight of the complete instrument to the size of the average portable radio set. Thermistor beads are available of sufficiently small diameter to fit in any but the hypodermic-size soil needles and sufficiently short to make the axial temperature gradient negligible.

The principal disadvantages of thermistors seem to be their fragility with respect to thermal and mechanical shock, and the necessity to check their calibration every few weeks if maximum accuracy is desired.

It has seemed to the groups both here and at the University of Western Ontario that these disadvantages are far outweighed by their convenience in use. Incidentally, we find that no precautions more stringent than those required in handling, say, a good microammeter are necessary to preserve the beads from mechanical damage, and the maximum safe temperature for a particular type is usually stated by the manufacturer.

The University of Western Ontario group feels that a conductivity accuracy of better than 5% and a diffusivity accuracy of better than 20% are obtainable with relatively crude apparatus, provided that the data are adequately treated. A little extra effort can reduce these relative errors by half, and with increased accuracy in temperature and current measurement it is hoped eventually to achieve accuracies of within 1% and 4%, respectively.

**Prof. E. F. M. van der Held** (*The Netherlands: communicated*): In Wageningen, the Netherlands (Agricultural University) researches on heat conduction of soil with different water contents have been done, both experimentally and theoretically. These confirmed the theory of Krischer on the influence of water upon the heat conduction.

Dr. de Vries deduced a formula for the heat conduction in granular materials which agreed with experimental data for water contents above 2% and for dry materials. Below 2% the experiments give lower values than the theory. In my opinion this deviation originates from the adsorptive forces which manifest themselves at layer thicknesses less than about 300Å. It would be interesting to compare the authors' data with the theories of Prof. Krischer and Dr. de Vries,* especially since, in their formulae, there are only measurable quantities.

## THE AUTHORS' REPLY TO THE ABOVE DISCUSSION

**Messrs. M. W. Makowski** and **K. Mochlinski** (*in reply*): We agree with Mr. Lythgoe that the paper represents only an interim stage in the solution of the many problems of the thermal environment of direct-laid cables.

One of the aims of the paper was to provide those potentially interested in the value of thermal resistivity of the soil with fairly simple methods which would enable them to make measurements in much greater number than hitherto. The need for such measurements to be taken by supply engineers was strongly emphasized by Mr. Bennett and also by the economic arguments of Messrs. Barnes and Cox.

* Blackwell, J. H.: *Canadian Journal of Physics*, 1956, 34, p. 412.

* Krischer, O., and Rohnalter, H.: *V.D.I. Forschungsheft*, 1940, 402.
Peerlkamp, P. K.: *Mededelingen Landbouwhogeschool*, 1944, 47, p. 1.
De Vries, D. A.: *Transactions of the Fourth International Congress of Soil Science*, 1950, 2, p. 41.
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van Wijk, W. R., De Vries, D. A., and van Duin, R. H. A.: *Netherlands Journal of Agricultural Science*, 1953, 1, p. 1.
De Vries, D. A., van Duin, R. H. A.: *ibid.*, p. 27.
De Vries, D. A.: *ibid.*, p. 115.
De Vries, D. A., and De Wit, C. T.: *Meteorologische Rundschau*, 1954, 7, p. 41.
De Vries, D. A.: 'The Thermal Conductivity of Granular Materials' (Institut International du Froid).

The best use of the two new methods is, as Dr. Konstantinowsky stated, in conjunction with each other; in this way a fuller knowledge of soil conditions can be achieved. We agree with Mr. Lythgoe that no method of measurement at present accepted should *a priori* be excluded. We think that each of the two rapid methods, as well as the sphere method, has a definite field of application for which the others are not suitable. Thus in non-homogeneous soil, when conditions permit a long-term steady-state investigation, the sphere (or a cylinder) has a decisive advantage because it averages thermal resistivity over a relatively large volume of soil. For a quick answer, in soft homogeneous soil the needle-probe method has an indisputable advantage over the other two. In the case raised by Mr. Cox, of soil in which other services operate and there is a possibility of the existence of thermal gradient, the sampling method is the only one which could be used safely.

A few speakers have mentioned the difficulty of dealing with soil containing a large amount of stone. We hope that, in the near future, it will be possible to extend the sampling method to such soils. If a needle probe cannot be directly inserted and time or resources do not permit the use of the sphere, some indication may be obtained by making an excavation or a narrow hole and inserting the needle in the sifted and punned soil, i.e. in the condition which would be encountered in cable practice.

Concluding the remarks on the application of the old and new methods, we certainly appreciate the usefulness of the sphere method in thermal-resistivity measurements, beside acknowledging its special application for diffusivity measurement mentioned by Dr. Griffith. Its main disadvantage is the cost and time required for a measurement. It can detect movement of moisture in soils, but the indications are only of limited application to cables, because of the difference of thermal gradient for the comparable dimensions and temperatures.

Lack of comparative data between the sphere and needle methods in the paper is due to the fact that possible discrepancies between measurements by the two methods are more a question of application than of the already proved principle. We think that, unless great care is taken, the sphere in its application tends to have a bigger contact resistance with the soil, and thus the readings are higher than if taken with the needle.

Referring to the points on the needle-method technique, we welcome the standardizing arrangement described by Mr. Hole. For temperature measurement the increase in number of thermocouples tends to decrease possible errors of the measurement. The resistance method of measurement, mentioned by Mr. Axworthy, whether by coil or thermistor can simplify testing, but must be carefully applied (see Section 3.3.3).

The need for more robust needles seems to be best met by the use of stronger materials. Strengthening with filler brings the disadvantage of increased thermal capacity. Stainless steel has been successfully tried at the E.R.A. and elsewhere. It has the advantage of lower thermal conductivity than either brass or Dural, so that a lower ratio of $L/d$ can be used. The ratio of 100 : 1 has been based on published data of previous investigators; it has not been checked thoroughly, but during tests on Dural tubes at a length/diameter ratio of about 60 : 1, the temperature at the tube centre was found to be marginally affected by end effect. The deviation could probably be compensated theoretically, but the effect would no doubt be increased if the experimental probe as used in the tests had the head which has of necessity to be used for probes in practice. We think the accuracies quoted by Dr. Blackwell in his valuable contribution are too stringent to be expected in practical outdoor conditions. His work on this subject should, however, be studied with interest.

In reply to Dr. Busemann's question about the inverted curvature of the heating curve, this is mainly due to the fact that the thermal properties of the probe are different from those of the soil. The problem is complex; arrangements giving the other curvature, e.g. using fillers, have been tried but they have no known advantages.

We are pleased that Dr. Kersten and Dr. Gemant think that our paper is a useful continuation of their work. Our aim was to relate their findings more strictly to the value of the clay content.

Dr. Kersten ascribes the discrepancy between direct measurement and curves based on the formula for Sandy Lodge (Fig. 10) to insufficient evaluation of dry density. The figure of 90 lb/ft$^3$ is, of course, the average of several readings. The recent measurements by needle and sampling methods show better agreement with Gemant's curves. The points obtained by the sphere method give large dispersion, but even there the correlation lines, especially of the second order, show good agreement. The curves of Fig. 10, mentioned also by Dr. Konstantinowsky, are derived from Kersten's empirical results and Gemant's theoretical assumptions. The measured values give better agreement with the curve corresponding to $\rho_d = 95$ lb/ft$^3$. In the paper we stressed the difficulty of measurement in dry sand. In general, the dry-density evaluation shows great dispersion. Repeated measurements in close proximity have given the following values for $\rho_d$: 88·7, 91·1, 94·0 and 84·8 lb/ft$^3$.

We agree with Mr. Fink on the value of statistical methods, and that the use of the tension table may be more expedient for the evaluation of minimum possible moisture content in a given soil than continued direct measurement. Unfortunately, the use of back scattering of neutron and $\gamma$-rays for $m$ and $\rho_d$ evaluation is not yet sufficiently established to be recommended for routine measurements.

The sampling method may, in the near future, be developed to permit application of the nomogram to a wider range of soils (e.g. chalky soils) and to soils containing a substantial proportion of stone.

The question of moisture migration under thermal gradient raised by several speakers, deserves special attention and particularly the interesting suggestion by Prof. Hooper, but it lies outside the scope of the paper, and therefore is deliberately excluded from consideration.

Both rapid methods discussed in the paper are based on the assumption of static and homogeneous conditions in the soil undergoing measurement. The physical quantity of thermal resistivity or conductivity in soil, for the determination of which the methods were set, is a steady-state quantity used primarily to solve steady-state problems, the corresponding dynamic quantity being thermal diffusivity.

Effects of moisture movement under thermal gradient are now being intensively studied by several investigators, and it is hoped that in the not too distant future the problem will be satisfactorily solved.

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# TEMPERATURE RISES IN ELECTRICAL MACHINES AS RELATED TO THE PROPERTIES OF THERMAL NETWORKS

By J. J. BATES, Ph.D., Graduate, and Professor A. TUSTIN, M.Sc., Member.

(*The paper was first received* 24*th August, and in revised form* 21*st December*, 1955. *It was published in April*, 1956, *and was read before the* UTILIZATION SECTION 19*th April*, 1956.)

## SUMMARY

The conditions are reviewed under which an electrical machine may usefully be regarded, for the purpose of relating temperature rises to losses, as a thermal network of lumped conductances and capacitances, having an electrical analogue.

Although the thermal resistivities are distributed, it is shown that lumped-resistance networks may be found that reproduce the relationships between the inputs of heat to the parts and the mean temperature rises over the volumes of the parts. Sources of non-linearity are considered, and it is shown that in many cases a linear model is adequate. The effect of transport of heat by the flow of a cooling medium cannot be introduced into a simple network model. The equations are linear, but do not have the properties of symmetry and reciprocity that are characteristic of static networks. Examples of such equations are given and some comments are made on the general form of the relationship between temperature rises and heat inputs, and the uses of such relationships.

Consideration is given to methods of testing to discover the coefficients of such equations, or the constants of an analogous electrical network where this exists, and methods that make use of intermittent loading are described. A novel timing device for use in such testing, a novel slip-ring contact for use with temperature detectors on rotating parts, and some special features of resistance-type temperature detectors are described.

The results of measurements on a 25 h.p. mill-type motor are given both for forced-ventilated and for totally-enclosed conditions, and the appropriate equations, and where possible the equivalent thermal networks, are deduced. The measurements also include information about the temperature distribution in various parts, both for continuous and for variable loading.

One purpose of the work was to provide a basis for a new approach to the problem of the estimation of temperature rises with variable loads and speeds. The ground-work for this is included, but the specific problem is discussed in a separate paper.

## (1) CONDITIONS UNDER WHICH AN ELECTRICAL MACHINE MAY BE REGARDED AS A THERMAL NETWORK

The limitations to the temperature rises to be allowed in electrical machines relate to parts such as insulation, brush contact surfaces, lubricants and soldered joints. The temperature rises of different parts may be very unequal, especially on intermittent loading and on highly cooled machines. The temperature rise caused by a unit of heat input may be very different according to where this input occurs.

The attempt to distinguish the various active parts, each with its particular input of heat, leads immediately to the observation that the structure approximates to a thermal network. If the parts in which heat is generated had infinite thermal conductivity, and were separated from each other and the cooling medium by layers of insulation or surfaces across which heat transfer was proportional to temperature difference, then the active parts would be the nodes of a thermal network, in which the ambient or cooling medium is a node that acts as a sink.

In so far as this is the case, and the parameters of the network can be evaluated, all the theorems and procedures of network analysis, including the various operational methods, may be brought to bear on thermal problems. Harmonic analysis may be used to deal with periodic variations, and observations on electrical network analogues may be used as an alternative to computation.

Electrical machines and other types of apparatus differ in certain features from the conceptual model that would have a linear lumped-constant electrical network as its exact analogue. A first purpose of this paper is to examine these differences.

### (1.1) Effect of the Distribution of some of the Thermal Resistances as Volume Resistivities in the Active Parts

The active parts would have uniform temperatures only, in general, if their thermal conductivities were infinite. Then the thermal resistivities would be confined to the inert parts such as the insulating material separating the active parts or resistances at their surfaces of contact with the cooling medium. Under these conditions the equations relating the temperature rises and the heat inputs to the various parts could be expressed in terms of 'coefficients of conductance', whatever the shape and inhomogeneity of the inert parts and the non-uniformity of surface resistances might be. This follows from well-known principles analogous to those by which inter-capacitances are defined for perfect electrical conductors separated by non-homogeneous dielectrics. In this idealized case, an equivalent lumped network would exist, whose parameters could be discovered by measurement. It is not obvious that an analogous lumped network exists when the thermal resistivities to a significant extent include volume resistivities of the active parts themselves. The temperature rises of these parts will not then be uniform, and will depend on the distribution of the generation of heat over the volume of the parts.

Consider, for example, the armature winding of an electrical machine. There may be considerable differences of temperature over the end-windings and the embedded length. One might attempt to take account of this by regarding the two end-windings and the embedded part as three distinct 'parts' from the thermal point of view. But in this case there is a large thermal conductance along the conductor between the end-winding part and the embedded part, which is entirely distributed in the volumes of these parts.

It may be shown, however, for example by the simple proof given in Appendix 14.1, that such distributed resistivities do not prevent the existence of a lumped network equivalent to the actual structure for steady-state conditions.

It is shown that, if the heat input over each of any arbitrarily defined parts is uniform over their volume, a lumped network exists in which the node temperatures are the mean temperatures of the parts and the inputs of heat to the nodes are the total inputs of heat to the parts.

If the input of heat is distributed non-uniformly but in fixed

Dr. Bates, formerly at Birmingham University, is at the Military College of Science.
Prof. Tustin, formerly at Birmingham University, is Professor of Electrical Engineering at the Imperial College of Science and Technology, University of London.

proportions over the volumes considered, there is still a valid thermal network, but the mean temperatures of the parts must be understood as the mean of the local temperatures weighted according to the rates of heat input per unit volume in each locality. In other words, one should average the temperature rises associated with equal elements of heat input rather than equal elements of volume.

This theorem greatly extends the range of practical situations to which one may apply with confidence methods of analysis that are equivalent to the assumption of a lumped linear thermal network.

### (1.2) Non-linearity of Relationship between Heat Transfer and Temperature Difference in Certain Cases

There are some exceptions to the general rule that the rate of heat transfer is directly proportional to the temperature difference.

The transfer of heat by radiation depends upon the difference between the fourth powers of the absolute temperatures of the parts, and linearity must not be assumed where radiation plays a significant part. This occurs only when temperature differences are large, and it is found that the non-linearity associated with radiation is rarely of importance in machines. The rate of heat transfer by radiation is given by

$$K\{(\theta_1 + 273)^4 - (\theta_0 + 273)^4\} \quad . \quad . \quad . \quad (1)$$

where $K$ is a constant depending on the character of the surface, $\theta_1$ is the temperature of the surface, and $\theta_0$ that of its surroundings, in degrees centigrade. Fig. 1 shows the values of $K(\theta + 273)^4$ for a representative value of $K$, and chords between points on this curve have slopes that correspond to the rate of heat transfer per square centimetre per degree difference. The slopes of the two chords shown in Fig. 1, for example, indicate the heat

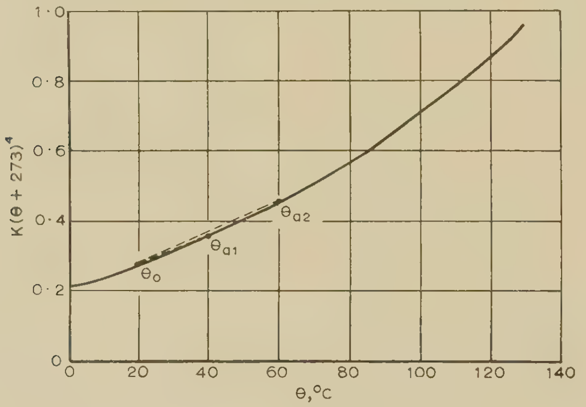


**Fig. 1.**—Variation with temperature of heat transfer by radiation. For the particular case of $K = 1$, i.e. unit emissivity.

transfer coefficients that would apply on account of the radiation component for the outer surface of a machine at 40° and 60° C respectively, with the ambient medium at 20° C. As this is only part of the total dissipation, the difference between the slopes is not large, and the temperatures involved are not widely variable, it is permissible to include radiation in a coefficient of dissipation or equivalent conductance that is regarded as a constant.

Another form of heat transfer in which the rate of transfer is not proportional to temperature difference is in cooling that involves natural convection, because the hotter the part the faster is the air movement produced and the greater the effective conductance. For a horizontal cylinder the rate of heat dissipation by natural convection is approximately proportional to the $\frac{5}{4}$ power of the difference of temperature between the surfac[e] and the surrounding air. Here again the departure from non[-]linearity is important only if temperatures are in question tha[t] have values over a wide range.

In the great majority of cases, especially in rotating electrica[l] machines, these effects do not introduce significant error i[n] applying coefficients of conductance that have been empiricall[y] determined and assuming them to be constant.

### (2) EQUATIONS FOR TEMPERATURE RISE TAKING INT[O] ACCOUNT THE TRANSPORT OF HEAT BY THE COOLIN[G] MEDIUM

Although the unidirectional effects of the flow of coolin[g] medium prevent exact analogy with a simple network, they d[o] not greatly complicate the equations for the steady state, whic[h] remain linear. In Fig. 2 the case is represented in which [a]

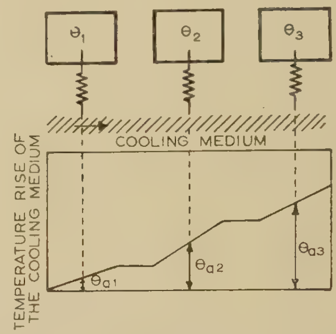


**Fig. 2.**—Temperature rises of a cooling medium flowing over a numbe[r] of parts.

stream of cooling medium receives heat successively from sever[al] parts, the thermal conductances from the successive parts bein[g] $K_{11}$, $K_{22}$ . . ., etc. The temperature rise of the stream of coolin[g] medium is $1/C$ degrees centigrade per watt of heat absorbe[d] where $C$ is the thermal capacity, in joules, of the quantity passin[g] per second. Let the temperature rises of the stream adjacen[t] to the several parts be $\theta_{a1}$, $\theta_{a2}$ . . ., etc. These temperature rise[s] are related to and determined by the temperature rises of th[e] parts, $\theta_1$, $\theta_2$ . . ., above the initial temperature of the coolin[g] medium by the following equations, on the assumption that th[e] effective temperature rises of the cooling medium, $\theta_{an}$, adjacen[t] to the $n$th part may be taken as the rise after absorbing half o[f] the dissipation from part $n$ in addition to the heat from th[e] preceding parts.

For heat balance in the cooling medium at successive points,

$$\theta_{a1} = \frac{K_{11}(\theta_1 - \theta_{a1})}{2C}$$

$$\theta_{a2} = \frac{K_{22}(\theta_2 - \theta_{a2})}{2C} + \frac{K_{11}(\theta_1 - \theta_{a1})}{2C} + \theta_{a1} \quad . \quad (2)$$

$$\theta_{a3} = \frac{K_{33}(\theta_3 - \theta_{a3})}{2C} + \frac{K_{22}(\theta_2 - \theta_{a2})}{2C} + \theta_{a2}$$

etc.

These equations relate the temperature rises at various points i[n] the air stream to the temperatures of the parts, and together wit[h] the equations for heat balance for the individual parts, the[y] suffice to determine the steady-state temperature distribution fo[r] a given distribution of losses. If $K_{12}$ and $K_{13}$ are the conductance[s] between parts 1 and 2, 1 and 3, etc., and $W_1$, $W_2$, $W_3$ are th[e] rates of heat input to the parts (considering three parts fo[r] simplicity) the equations for heat balance for the three parts ar[e]

$$W_1 = K_{11}(\theta_1 - \theta_{a1}) + K_{12}(\theta_1 - \theta_2) + K_{13}(\theta_1 - \theta_3)$$
$$W_2 = K_{21}(\theta_2 - \theta_1) + K_{22}(\theta_2 - \theta_{a2}) + K_{23}(\theta_2 - \theta_3)$$
$$W_3 = K_{31}(\theta_3 - \theta_1) + K_{32}(\theta_3 - \theta_2) + K_{33}(\theta_3 - \theta_{a3}) \quad (3)$$

Together with the equations for heat balance at successive points in the cooling medium, these are sufficient in number to determine all the temperatures, and the complete system is linear.

If the temperature of the cooling medium is effectively the same adjacent to all the parts and is $\theta_a$, one may redefine $\theta_1, \theta_2 \ldots$ as temperature rises above the temperature of the cooling medium, and then the equations for heat balance are simply

$$W_1 = K_{11}\theta_1 + K_{12}(\theta_1 - \theta_2) + K_{13}(\theta_1 - \theta_3)$$
$$W_2 = K_{21}(\theta_2 - \theta_1) + K_{22}\theta_2 + K_{23}(\theta_2 - \theta_3)$$
$$W_3 = K_{31}(\theta_3 - \theta_1) + K_{32}(\theta_3 - \theta_2) + K_{33}\theta_3 \quad . \quad . \quad (4)$$

These are the familiar equations of a linear network, in which the conductances between parts have the same values for flow in either direction, i.e. $K_{12} = K_{21}$, etc., so that the coefficients are symmetrical about the diagonal. This results in the property of reciprocity, one form of which is that the temperature rise of any part A per watt in part B is the same as the temperature rise of part B per watt in part A.

The general properties of linear networks all apply in this case and they need not be discussed here, except that in Appendix 14.2 a simple development is given of a relationship that is particularly useful in thermal problems, namely

$$\theta_1 = \frac{W_1 + B_{21}W_2 + B_{31}W_3 + \ldots}{K_{11} + B_{21}K_{22} + B_{31}K_{33} + \ldots} \quad . \quad . \quad (5)$$

with analogous expressions for $\theta_2$, $\theta_3$, etc., which express the temperature rises as a weighted sum of the inputs of heat divided by a similarly weighted sum of the conductances from the several parts to the cooling medium. The weighting factors $B_{21}$, $B_{31}$, etc., in the expression for $\theta_1$ are the ratios of the temperature rises of parts 2, 3, etc., to the temperature rise of part 1 when there is heat input to part 1 only.

It is desired here only to make the observation that these properties of symmetry and reciprocity are no longer present when the varying temperature of the cooling medium, increasing in the direction of flow, is taken into account by including the set of equations for heat balance in the stream. The system is linear, but the equations have no longer the diagonal symmetry that is characteristic of the equations of a linear network. Exact

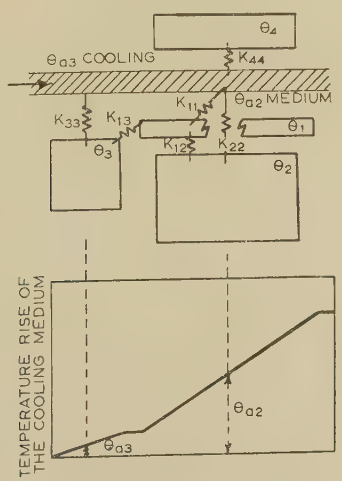


**Fig. 3.**—Thermal scheme for a ventilated motor.

representation by a network of simple conductances is no longer possible, though it may be justifiable in some cases as an approximation. Results of measurements will be given later which show that the departure from reciprocity may be considerable. When solutions for the complete set of equations are required, they are best obtained for specific applications, particularly as the cooling-stream relationships are often different from such a scheme as Fig. 2. For example, in a self-ventilated motor with the air intake at the commutator end, it is evident that the air reaching the commutator will be at ambient temperature, but if the remaining parts distinguished are the armature winding, core and field coils, these are all located with their centre-lines coincident, and the air is heated over this neighbourhood by the dissipation from all of them. The situation is better represented by the scheme of Fig. 3, which gives the supplementary equations

$$\theta_{a3} = \frac{K_{33}(\theta_3 - \theta_{a3})}{2C}$$

$$\theta_{a2} = \frac{K_{22}(\theta_2 - \theta_{a2}) + K_{11}(\theta_1 - \theta_{a2}) + K_{44}(\theta_4 - \theta_{a2})}{2C} + 2\theta_{a3} \quad (6)$$

The relations between temperature rises and losses, whilst not usually those of a network, are thus likely to be linear, so that the property of superposition may be presumed. Thus the temperature rise of a part is always the sum of components proportional to the losses in the several parts,

i.e. $$\theta_n = f_{1n}W_1 + f_{2n}W_2 + \ldots \quad . \quad . \quad . \quad . \quad (7)$$

where the coefficients $f$ may be found by experiment. These have the dimensions of thermal resistance and are functions of the conductances $K$ and the heat-transport coefficient $C$ only.

A second useful observation is that, for any period over which the change of temperature and so the change of stored heat between the beginning and end of the period is negligible compared with the dissipation of heat over the period, the equations that relate the average temperatures of the parts to the average rate of input of heat are exactly the same as those for steady-state conditions. Thus average temperatures may be found from the same data as steady-state temperatures, and this provides a basis for dealing with sustained variable loadings.

The designer, as distinct from the user, is interested in other and more detailed aspects of the relationships. A knowledge of the coefficients $f$ in equations such as (7) suffices to determine the temperature rises on continuous ratings, but only subject to the error associated with the determination of the losses in various parts. The separation of losses has difficulties that are well known. Nevertheless, this is no reason for incurring the further gross errors that arise when an attempt is made to determine the temperature rises of specific parts, e.g. of an armature winding, from the total loss and some coefficient related to the overall dimensions. It is shown in a companion paper how these relationships make it possible to deduce temperature rises in service directly from temperatures observed on test, without explicit reference to the amount of the losses.

To the designer an even more important use of the detailed analysis is to estimate the value of various possible modifications intended to increase output. For this purpose the evaluation of the separate thermal conductances from and between the various parts [coefficients $K$ of eqn. (3)] is most valuable. Particular conductances may be examined in the light of physical data such as the thermal conductivities of materials, and possibilities of improved heat dissipation are likely to appear in the course of such studies.

## (3) DERIVATION OF HEATING COEFFICIENTS OR OF PARAMETERS OF AN EQUIVALENT NETWORK FOR ELECTRICAL MACHINES FROM RESULTS OF TESTS

Relationships of temperature rise to losses having various degrees of precision may be observed by postulating systems of equations of various degrees of detail, and obtaining the coefficients from measurements of the temperature rises for known losses.

In an equation system for $n$ parts in the form

$$\begin{aligned} \theta_1 &= f_{11}W_1 + f_{21}W_2 + \dots f_{n1}W_n \\ \theta_2 &= f_{12}W_1 + f_{22}W_2 + \dots f_{n2}W_n \\ &\;\;\vdots \\ \theta_n &= f_{1n}W_1 + f_{2n}W_2 + \dots f_{nn}W_n \end{aligned} \quad \dots \quad (8)$$

which makes no assumptions except that of linearity, the $n^2$ coefficients may in principle all be determined from measurements of $n$ temperature rises on each of $n$ tests. The $n$ coefficients for $\theta_1$, for example, could be obtained by solving the $n$ equations relating the measured values of $\theta_1$ to $n$ different combinations of $W_1, W_2 \dots W_n$. The results are well determined only if the loss distributions are sufficiently distinctive, and for each set of coefficients required the labour of solving a set of simultaneous linear equations is involved. This labour is reduced if some of the losses are zero in particular tests.

It would be a great convenience if such measurements could be made with heat inputs to only one part on each test. Not only are the distributions then the most distinctive possible, but the equations to determine the coefficients for $\theta_1$, for example, become

$$\begin{aligned} \theta_{11} &= f_{11}W_1 \\ \theta_{21} &= f_{21}W_2 \\ \theta_{31} &= f_{31}W_3 \\ &\text{etc.} \end{aligned} \quad \dots \dots \quad (9)$$

and the coefficients for $\theta_2$ are similarly given from the same tests by

$$\begin{aligned} \theta_{12} &= f_{12}W_1 \\ \theta_{22} &= f_{22}W_2 \\ \theta_{32} &= f_{32}W_3 \\ &\text{etc.} \end{aligned} \quad \dots \dots \quad (10)$$

If the loss cannot be confined to a single part there is advantage and simplification in confining it to only two parts or as few parts as possible. In a d.c. motor, the principal distinct active parts are the commutator, the armature winding, the armature core and the field windings. It is not obvious how tests for steady-state temperature rises may be contrived in which only one of these parts is subject to loss, the cooling being that corresponding to a specified speed. For example, the passage of current through the armature winding with the machine rotating involves losses in core and commutator as well as in the winding.

A procedure that may be used to avoid this difficulty is to make the heating test intermittent. For constant conductances it makes no difference to the equations for heat balance if instead of the heat input occurring continuously it occurs intermittently but at the same mean rate, provided that the mean temperatures have become steady and that the temperatures measured are the mean temperatures. For varying conductances there is some possibility of error due to correlation between conductance and temperature variations. If the conductances tend to be high when the temperatures are high and low when these are low, there is a greater dissipation of heat than is given by the products of mean conductances and mean temperature rises. Provided the temperature variations are fairly small, however, the error is negligible as is shown in Appendix 14.3.

For example, if current is passed through the armature winding for a short time with the armature at rest, or rotating just fast enough to avoid local overheating of some commutator bars, there will be no core or eddy-current losses and no brush friction, provided that the brushes are lifted during the current-off period. The total losses are the $I^2R$ loss together with the brush-drop loss. If then the armature is immediately run for a further period without current and at a speed whose mean is that for which the data are required, and if this operation is repeated regularly with periods of a few minutes until all the temperature rises have become steady, the conditions can be secured that permit of easy analysis. The important and possible doubtful assumption involved in this simple procedure is that variation of conductances with speed are sufficiently linear to permit identification of the mean conductances with the conductances at the mean speed. This will be dealt with more fully later, suggestions being made for allowing for the possible effect of such non-linearity.

One may analogously obtain loss in the exciting (field) winding without core loss and with a required mean speed by applying a high current through the exciting winding with the armature stationary, and then running the armature for a period at a suitable speed to give the required mean speed, this cycle being repeated regularly until the mean temperatures cease to change.

## (4) MEASUREMENTS OF HEATING COEFFICIENTS FOR A 25 H.P. MILL-TYPE MOTOR

The tests shown in Table 1 were carried out on a type-60 mill-type d.c. series motor, forced ventilated. The motor was a standard motor rated at 25 h.p. when totally enclosed.

**Table 1**

TESTS ON A FORCED-VENTILATED MOTOR

| Test No. | Method of test | Losses confined to |
|---|---|---|
| 1 | Alternately a high current at 10 r.p.m. for 3 min followed by required higher speed, brushes lifted | Armature winding and commutator |
| 2 | Continuous, no load, no excitation, brushes down | Commutator |
| 3 | Alternately, current in exciting winding with motor rotated at 10 r.p.m. followed by required higher speed, brushes lifted | Exciting winding |
| 4 | Continuous with exciting current, at required speed | Exciting winding and core |

In each case the mean speed was the same, namely 590 r.p.m. and the losses were measured and the mean temperatures recorded after the temperatures had attained a steady mean. This was done by measuring the temperatures on terminating the test at the mid-point of a period, and also, in this case, since the machine was fitted with embedded resistive temperature detectors, by continuous observations.

By substituting the observed temperature rises and the mean losses in the four equations of the type (8) and solving these for the heating coefficients $f$, the values shown in Table 2 were obtained.

If the relationships had been those of a network, these coefficients would necessarily have been symmetrical about the leading diagonal, since such symmetry expresses the property

**Table 2**

HEATING COEFFICIENTS FOR A FORCED-VENTILATED MOTOR

| Temperature rise of | Due to unit loss in | | | |
|---|---|---|---|---|
| | Armature winding | Core | Commutator | Exciting (field) winding |
| Armature winding | $f_{11} = 0{\cdot}045$ | $f_{21} = 0{\cdot}027$ | $f_{31} = 0{\cdot}032$ | $f_{41} = 0{\cdot}007$ |
| Core .. | $f_{12} = 0{\cdot}038$ | $f_{22} = 0{\cdot}033$ | $f_{32} = 0{\cdot}023$ | $f_{42} = 0{\cdot}006$ |
| Commutator | $f_{13} = 0{\cdot}017$ | $f_{23} = 0{\cdot}013$ | $f_{33} = 0{\cdot}083$ | $f_{43} = 0{\cdot}003$ |
| Exciting (field) winding | $f_{14} = 0{\cdot}011$ | $f_{24} = 0{\cdot}010$ | $f_{34} = 0{\cdot}006$ | $f_{44} = 0{\cdot}070$ |

Heating coefficients are in degrees centigrade per watt.

of reciprocity. The departure from equality of corresponding coefficients is considerable. The unidirectional transport of heat by the ventilating air may account for this dissymmetry.

## (5) VARIATION OF TEMPERATURE-RISE COEFFICIENTS WITH SPEED

Similar tests were made at various speeds and the coefficients calculated. At the lower speeds this involved a certain amount of extrapolation in some cases, as certain losses are small at low speeds; for example, the temperature rises due to core loss become too small to be accurately determined. The best estimates that could be made of the coefficients are recorded as functions of speed in Figs. 4, 5, 6 and 7. Fig. 4 gives the temperature rises per unit of armature winding loss as directly recorded in the intermittent tests with a loss mainly in the armature winding but with some small commutator loss, and is presented in this form to display the consistency of the measured

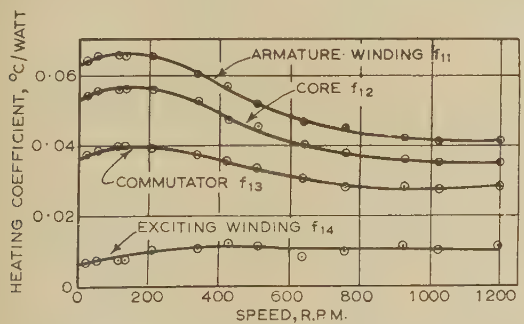


Fig. 4.—Coefficients for temperature rises due to loss in armature winding: no allowance for commutator loss.

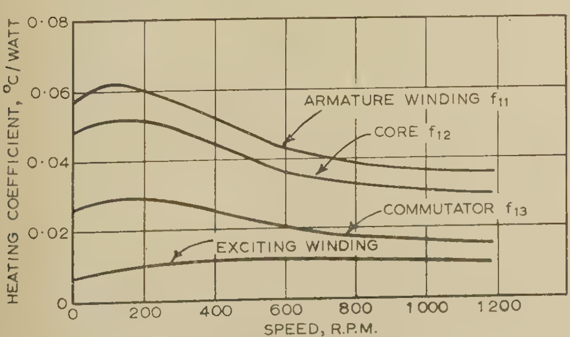


Fig. 5.—Coefficients for temperature rises due to loss in armature winding: corrected for commutator loss.

values. Fig. 5 gives the corrected heating coefficients after subtracting the contribution of the commutator loss to the temperature rises. Some extrapolation was necessary in this correction as the coefficients for the effect of loss at the commutator, $f_{31}$, $f_{32}$, $f_{33}$, necessary for the correction, could not be found directly at low speeds.

An unexpected feature of these curves, which was confirmed by additional measurements as shown by the points recorded

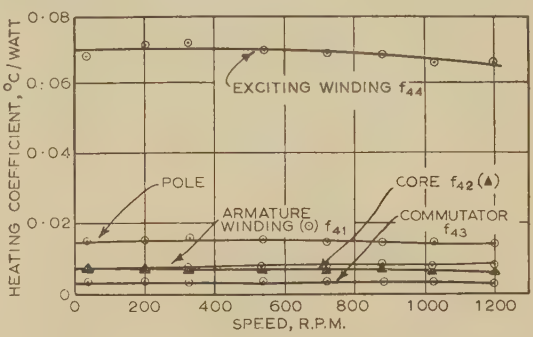


Fig. 6.—Coefficients for temperature rises due to loss in exciting winding.

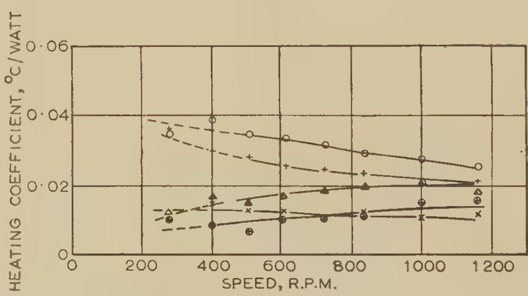


Fig. 7.—Coefficients for temperature rise due to core loss.

○ ○ Core, $f_{22}$.
+ + Armature winding, $f_{21}$.
△ Pole.
× × Commutator, $f_{23}$.
⊗ ⊗ Exciting winding, $f_{24}$.

on Fig. 4, is that in the lower range of speeds, below 400 r.p.m. and particularly below 200 r.p.m., the temperature rises per watt decreased as the speed was reduced, whereas one would expect that the conductances would continue to decrease and the temperature rises per watt to increase.

It must be remembered that the motor was forced ventilated. The ventilation was nominally kept constant by keeping the speed of the blower constant. The fact that the air volume did not vary significantly was verified by the observed constancy of air outlet temperature rise per watt of loss. The cause of the increased cooling as zero speed is approached may be that the arrangement of the motor fan, which is shown in the sectional view of the motor, Fig. 8, is such that, in the air paths over the armature surface, the fan head opposes the head applied externally by the blower, and at normal speeds the flow of air through the outer parts of the machine is reduced on this account while the flow through the internal ducts is increased. At sufficiently low speeds this effect is insignificant, and the air in the gap and interpolar spaces has a relatively higher speed, so accounting for the reduction in the heating coefficients at low speeds in place of the increase that would otherwise occur. This explanation was confirmed by observations of the temperature rises of the air stream at the points A, B, C and D shown in Fig. 8, the results of the measurements being given in Fig. 9. These show that between 0 and 500 r.p.m. there is an increase of some five times in the temperature rise of the air flowing over the armature

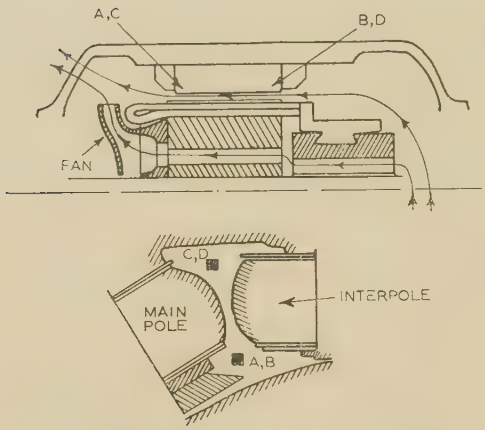


**Fig. 8.**—Flow of ventilating air and location of temperature detectors in air stream.

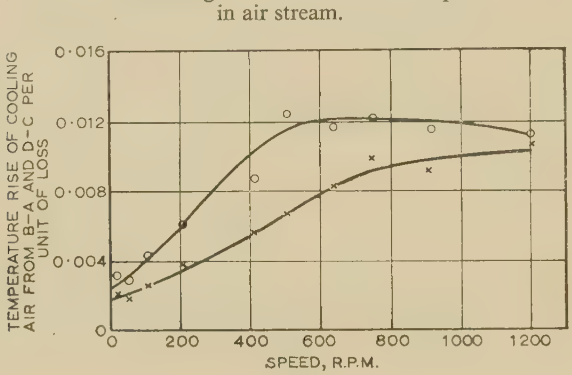


**Fig. 9.**—Rise in temperature of air stream per watt loss as measured by temperature detectors in positions shown on Fig. 8.

O—O *D*—C.
×—× B—A.

for the same loss in the armature winding, which indicates a slower movement of air over the outer surface of the armature. It suggests a possibility of improvement in the design of this particular motor if it is to be used with forced ventilation, since the implication is that, if this defect in ventilation were absent, the curves giving the heating coefficients would be lowered at all speeds except quite near zero.

## (6) EFFECT OF A NON-LINEAR RELATIONSHIP BETWEEN CONDUCTANCE AND SPEED ON INTERPRETATION OF INTERMITTENT TESTS FOR DETERMINING HEATING COEFFICIENTS

### (6.1) The Simplicity of Interpretation with a Linear Relationship

It was expected, when the measurements were planned, that they would reveal an approximately straight-line relation between heat dissipation and armature speed, and the interpretation of the results of intermittent tests is simplest if this may be assumed, since then the mean heat dissipation must be the same as that at the mean speed. The curves giving heating coefficients have been plotted as functions of mean speed, i.e. the mean speeds actually used in the intermittent tests. One would like to be able to say that these coefficients would be the same whenever the mean speed had a given value, irrespective of how the means were made up of periods of high and low speed, and that the coefficients as recorded would also be those for constant speeds. This is not exactly the case. It would be so if those conductances that varied with speed did so linearly, but the heating coefficients show a marked departure from linearity.

### (6.2) Method of Interpretation with a Non-Linear Relationshi

It is possible to deduce, from the test results, curves that show the dependence of the heating coefficients on speed, as when the speed is held constant, provided that, as was actually the case there is available a test in which the low speed used for the on-load period is continued unchanged over the off-load period.

Let $\gamma$ = Mean rate of heat dissipation from armature winding during intermittent test, per deg C rise.
$\alpha$ = Rate of heat dissipation during current-on period per deg C rise.
$\beta$ = Rate of heat dissipation during current-off period per deg C rise.
$T_1$, $T_2$, $T$ = Current-on, current-off, and total times respectively.

For heat balance

$$\gamma = \alpha\frac{T_1}{T} + \beta\frac{T_2}{T} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$\gamma$ is obtained directly as the ratio of the mean armature winding loss to the measured winding temperature rise. For the one test which was continuous, and for which the speed was the low speed of the current-on-period used in all the intermittent tests, $\beta = \alpha = \gamma$, and this value may be calculated. Then knowing $\alpha$, $\beta$ can be calculated for each test and plotted against the corresponding current-off speed. This will give very nearly the true value of the reciprocal of the heating coefficient for the various speeds. There is a possibility of small error if there is a change in the relative temperature-distribution with speed, but the measurements showed little such change. Fig. 10 gives the

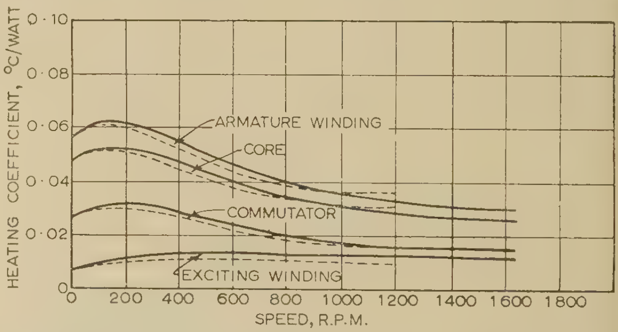


**Fig. 10.**—Coefficients for temperature rises due to loss in armature winding, allowing for possible non-linear variation of conductance with speed.

heating coefficient curves after correction in this manner, the broken curves being those of Fig. 5, where linearity was assumed. These more correct values alter the table of heating coefficients given previously. The values affected are those involving intermittent tests, i.e. the coefficients ($f_{11}$ to $f_{14}$) for heating due to armature-winding loss and those ($f_{41}$ to $f_{44}$) for heating due to exciting-winding loss, but the change of the coefficients with speed in the case of loss in the exciting winding (Fig. 6) is not significant. The corrected coefficients for the temperature rises due to loss in the armature winding (with uncorrected figures in brackets) are given in Table 3.

It will be noticed that in spite of the marked non-linearity (Fig. 5) the correction for non-linearity is small. Such intermittent tests should preferably be arranged to minimize any correction. For instance, the current-on time can be made very short consistent with adequate heating, say $T_1/T = 0{\cdot}1$. If also, for example, the conductances at the low speed during the current-on period were one-fifth of those at the higher speed during the current-off period, then all but 2% of the heat would

Table 3

HEATING COEFFICIENTS CORRECTED FOR NON-LINEAR VARIATION OF CONDUCTANCE WITH SPEED

| | | |
|---|---|---|
| $f_{11}$ | 0·047 | (0·045) |
| $f_{12}$ | 0·040 | (0·038) |
| $f_{13}$ | 0·020 | (0·017) |
| $f_{14}$ | 0·011 | (0·014) |

be dissipated at the higher speed, and the heating coefficient determined as the ratio of temperature rise to loss could be taken as that for the speed for the off-load period with little possibility of error.

### (6.3) Variation of 'Effective Conductance' with Speed

The reciprocal of the heating coefficient $\beta$ is an 'effective conductance' from the armature winding to the ambient medium. Fig. 11 shows the variation of this effective conductance with

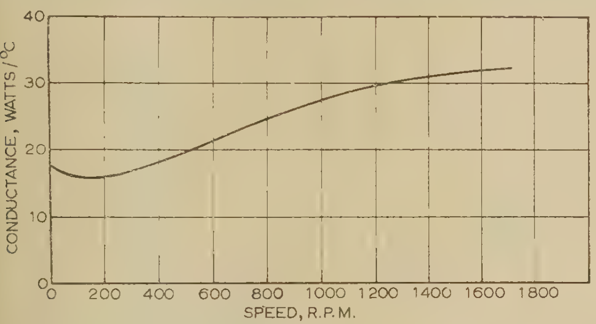


**Fig. 11.**—Equivalent conductance from armature winding to ambient medium.

rotational speed that corresponds to the heating coefficients of Fig. 10. It will be noticed that, in addition to the marked departure from linear dependence on speed at low speeds which has already been referred to, the remainder of the curve is concave downwards. This feature does not imply that the transfer of heat from the extreme surfaces to the ventilating air depends non-linearly on the surface speed. Each of the several channels of heat dissipation, from the armature copper to the ventilating air, is through thermal resistances in series, one part independent of speed and one part varying with speed, and such a combination, even if the conductance that varies with speed varies linearly, will give a curve of resultant conductance which tends to decrease in slope at high speeds.

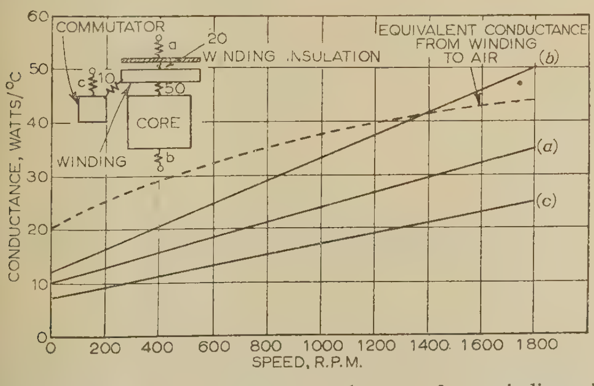


**Fig. 12.**—Variation of equivalent conductance from winding with an assumed linear variation of individual conductances.

Equivalent conductance from winding to air

$$= \frac{20a}{20+a} + \frac{50b}{50+b} + \frac{10c}{10+c}$$

To show that this feature suffices to account for the observed form of the curve of effective conductance at the higher speeds, the curves shown in Fig. 12 have been calculated for the thermal network that is also shown in the diagram, where the values of the thermal conductances are approximately those for the machine in question (see Section 7) but the conductances from the surfaces to the ambient medium are supposed to vary linearly with speed. The equivalent conductance from armature winding to ambient medium for this model, calculated as indicated in the diagram, varies as shown by the broken line, and this agrees closely in form with the effective conductance actually observed at the higher speeds.

A further analysis, possibly of less reliability on account of the small temperature-differences involved, is shown in Fig. 13,

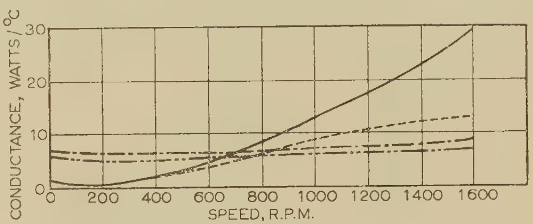


**Fig. 13.**—Estimated variation with speed of conductance from armature parts to cooling air.

——— Winding insulation—air.
— · — — Winding-copper—air (including path through winding insulation).
—— —— Core—air.
—— - - —— Commutator—air.

which gives an estimate of the separate conductances as functions of speed. This again anticipates the results of Section 7, in which a thermal network is derived for one particular speed. The thermal resistance of a square centimetre of the winding insulation was estimated from the conductance between winding and core and the total area of sides and bottoms of the slots. This same value was taken for the thermal resistance of the insulation over the exposed surface, and as the temperatures of the several parts and the adjacent air were measured, it is possible to estimate the several surface-air conductances at various speeds. This gave the results shown in Fig. 13, in which, again, the conductance from the surface of the insulation to the air is related to rotational speed nearly linearly over the higher range of speeds. The extremely low surface-air conductance in the neighbourhood of 200 r.p.m. will be noted; it is probably due to the exchange of air in the air-gap being interfered with by the opposition between the internal and external fans, as already suggested.

An unexpected result is the approximate constancy of the conductances from core and commutator, but this may well be possible in this particular motor. So far as the conductance from the core is concerned, it has already been explained that the fan reduces the air flow over the outer surface of the armature but increases it through the ducts. The combined effect may be to keep the core-air conductance nearly constant. The commutator was immediately above the air inlet at the bottom of the motor with the inlet air blowing directly onto it, so that turbulence would be very great whatever the rotational speed, and the dependence of this conductance on speed would therefore be expected to be slight.

Fig. 13 explains the flattening of Fig. 12 as being due to the inclusion of paths of constant resistance to heat flow in series with paths where the resistance changes with speed.

## (7) THERMAL NETWORK FOR FORCED-VENTILATED MOTOR

In the tests just described, the air temperatures adjacent to the various parts of the motor armature were measured as well

as those of the parts themselves. From these tests a sufficient number of simultaneous equations is available to calculate the conductances from and between the parts of the armature. The part of Fig. 14 below the block marked 'cooling medium' shows the thermal network representing the armature and the numerical values of the conductances found in this way.

The conductance from the exciting winding to the air stream was obtained from the results of tests made with loss in the exciting winding only, with the motor both totally enclosed and forced ventilated. The totally-enclosed test gives the power dissipated from the motor frame per degree centigrade rise of the frame above ambient. It may be assumed that when the motor is forced ventilated the dissipation from the frame is related to the frame temperature by the same coefficient. The remaining dissipation must be to the air stream. The conductance to the air stream is the dissipation per degree of temperature difference between the air stream and the exciting winding. The value found is also shown in Fig. 14.

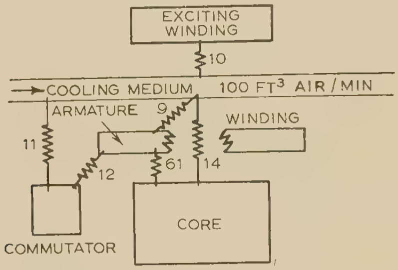


**Fig. 14.**—Thermal network for forced-ventilated motor.
Values are in watts per degree centigrade.

### (8) MEASUREMENTS WITH MOTOR TOTALLY ENCLOSED

A less extensive series of tests was carried out on the same motor when totally enclosed. Figs. 15 and 16 show the temperature rises per watt of loss in the armature winding and exciting winding respectively. Fig. 17 shows the coefficients for the effect of armature-winding loss corrected for possible non-linearity in the variation of conductance with speed, using the same method as for the forced-ventilated motor. The close agreement of the corrected curves with the uncorrected curves (broken) shows that the non-linearity must be small except at high speeds.

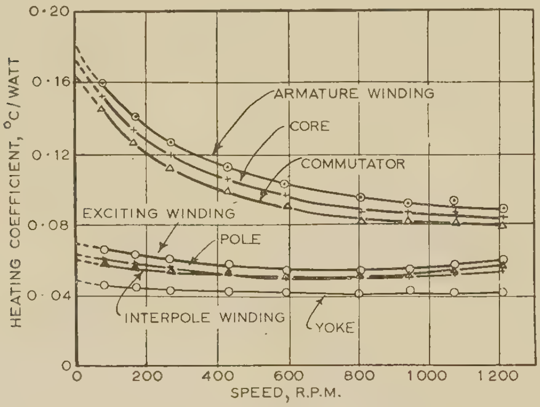


**Fig. 15.**—Coefficients for temperature rises due to loss in armature winding: totally-enclosed motor.

It is found, as would be expected, that under totally-enclosed conditions the relatively large thermal resistances between th armature and the frame and from the frame to the outer ai have a predominant role, and the effect of an element of loss i determined rather by whether it occurs in the armature or i the stator, with relatively small further differences according t the particular location within these parts. This suggests tha

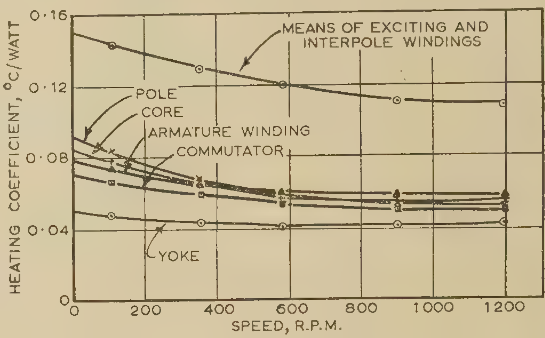


**Fig. 16.**—Coefficients for temperature rises due to loss in excitin winding: totally-enclosed motor.

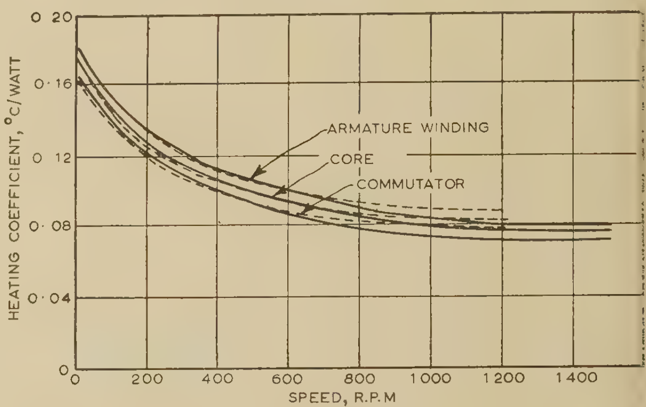


**Fig. 17.**—Coefficients for temperature rises due to loss in armatu winding, allowing for possible non-linear variation of conductan with speed: totally-enclosed motor.

for totally enclosed machines it may be adequate to lum together the parts of the armature and the parts of the stat respectively, and so to represent the machine as the simp network shown in Fig. 18. The conductances were measure

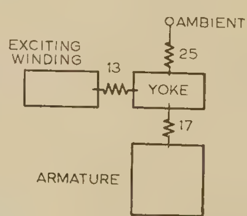


**Fig. 18.**—Thermal network for totally-enclosed motor.
Values are in watts per degree centigrade.

by recording the appropriate temperature differences for loss in the exciting (field) winding and armature winding in turn.

A striking result of the tests as indicated by Figs. 15 and is the close adherence to the principle of reciprocity, indicat by the exciting-winding rise per unit armature-winding loss nev differing by more than 6% from the armature-winding rise p unit exciting-winding loss, throughout the range of speeds.

## (9) TEMPERATURE GRADIENTS IN THE ARMATURE WINDING

Fig. 19 shows the different temperature rises per watt at three different points in the armature winding with the motor forced ventilated, as measured by three embedded detectors. These curves confirm that there is little temperature gradient in the windings, as would be expected on account of the high thermal

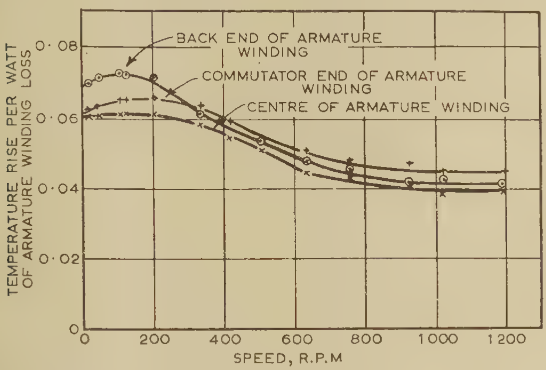


**Fig. 19.**—Temperatures at several points on armature per watt of armature loss: forced-ventilated motor.

○ Back end of armature winding.
+ Commutator end of armature winding.
× Centre of armature winding.

conductivity of copper. These results indicate that there is no necessity to consider the armature winding as more than a single thermal part.

In the totally-enclosed tests the temperature differences were barely measurable.

## (10) SOME DETAILS OF THE EXPERIMENTAL EQUIPMENT

### (10.1) Temperature Detectors

Resistance-type detectors were embedded in the main parts of the motor, care being taken that each detector was situated so that it gave an accurate indication of the temperature of the part in question. To measure the temperature of the embedded part of the winding in the slots, a temperature-sensitive resistance made by winding fine wire in the form of a very thin flat strip was developed to replace the normal spacer between top and bottom coil-sides in a slot. The construction of this detector and its location are shown in Fig. 20, which is self-explanatory.

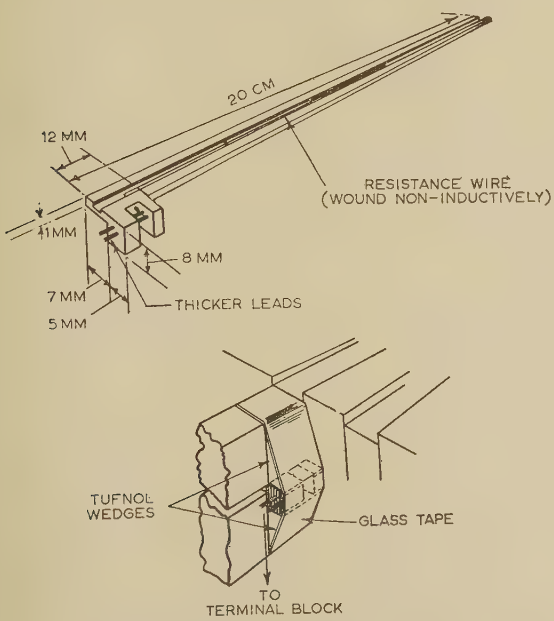


**Fig. 20.**—Embedded temperature detector for armature winding.

### (10.2) A Slip-Ring Contact Device for Use with Temperature Detectors on Rotating Parts

To avoid many of the difficulties associated with sliding contact between brushes and slip rings where measurement of small currents and voltages is involved, a novel type of contact device was developed that proved extremely reliable. The principle is shown in Fig. 21. The slip rings are in the form of

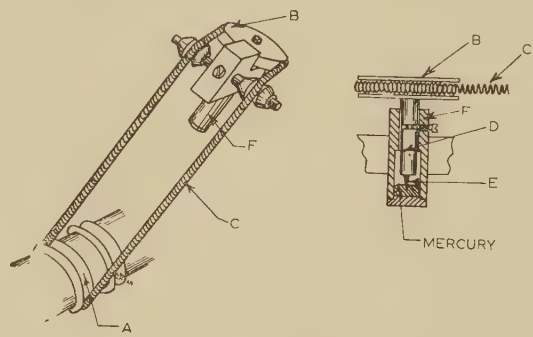


**Fig. 21.**—Novel type of electrical contact device.

A. Slip rings on motor shaft.
B. Top pulley.
C. Phosphor-bronze spring.
E. Stainless-steel needle.

pulleys (A) of phosphor-bronze with vee grooves. From each of these pulleys a drive is taken to a similar pulley (B) by means of a phosphor-bronze spiral or spring belt (C). The pulley (B) is mounted on a shaft (D) on the end of which is a stainless-steel needle (E). The shaft runs in a pot (F), in the bottom of which is a little mercury into which the needle dips. The stationary contact, or 'brush', is now the mercury, and the moving contact, or 'slip ring', is the needle point, and the relative velocity between these two is very small owing to the small diameter of the needle. The electrical contact between pulleys A and B occurs with a little slip of the phosphor-bronze spiral over the pulley surface, and this tends to keep the surfaces clean. With this contact device it was found possible to pass currents as small as a microampere without any observable variation with speed, the only resistance in circuit being that of the contact device itself.

### (10.3) A Relay Timing Device for Intermittent Tests

The principle is shown in Fig. 22. A uniselector U1 is pulsed at $\frac{1}{2}$sec intervals from a clock. Every time this uniselector completes its sweep a second uniselector U2 is stepped on by one step. By connecting one side of a relay, A, to a segment of one selector and the other side to a segment of the other

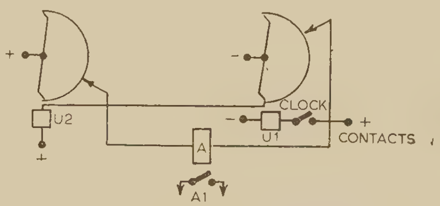


**Fig. 22.**—Relay timing device.

selector, the relay can be caused to operate at a particular chosen pulse of the clock up to a time equivalent to 625 pulses, as there are 25 segments on each selector. Any number of relays can be connected to the segments, so that any number of operations can be controlled within the cycle of 625 pulses. By bringing the uniselector segments to plug-and-socket boards, any chosen sequence of operations is speedily set up.

## (11) CONCLUSION

The analysis and the experimental results show that the relationship between losses and temperature rises in a machine running at a given speed may differ considerably from the relationships for a network, but that, for any given speed, the equations are linear. Means have been indicated for obtaining the coefficients of these equations by measurement, using on-off tests where necessary to obtain distinctive distributions of losses.

When the coefficients were measured in this way, and at various speeds, on a particular forced-ventilated motor, it was found, contrary to expectations, that the coefficients (conductances) were not linearly related to speed. The departure from linearity, in the case of this motor, was an impairment of cooling at all but low speeds. This was attributed to the arrangement of the fan on the particular motor.

The possibility that in some cases the relationship between the effective thermal conductances and the rotational speeds may be substantially non-linear is important because methods of estimating temperature rises in service for variable-speed operation are very simple if such linearity may be assumed. It appears to be necessary, in obtaining data for a machine, to make observations at several speeds and to use suitably modified procedures if marked non-linearity is found.

The principles and methods described have been illustrated in their application to d.c. mill-type motors, but they are applicable to a wide variety of electrical apparatus, including a.c. motors and transformers, for which analogous methods of using intermittent tests to obtain distinctive distributions of losses may readily be devised. It is hoped that the paper will lead to the exploration of these possibilities by designers in their various fields of interest.

## (12) ACKNOWLEDGMENTS

The authors gratefully acknowledge help given by The General Electric Company in lending a loading motor and in the installation of temperature detectors, and by Mr. J. Armstrong, whose services that Company made available to assist in some of the numerous tests. Thanks are also given to the British Iron and Steel Research Association for providing the mill motor, and for the bursary which enabled one of the authors to undertake the work described in the paper.

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## (14) APPENDICES

### (14.1) Proof of Existence of Lumped Thermal Network Equivalent to a System including Distributed Volume-Resistivities

In Fig. 23, the cubic mesh indicates a fine three-dimensional lattice of resistances, so distributed as to correspond to the actual distributed resistivities. The lattice need not be cubic; its precise form is immaterial. The regions marked A, B, etc. represent the 'active' parts, i.e. parts in which loss occurs. It will be supposed that within any one of these regions there are

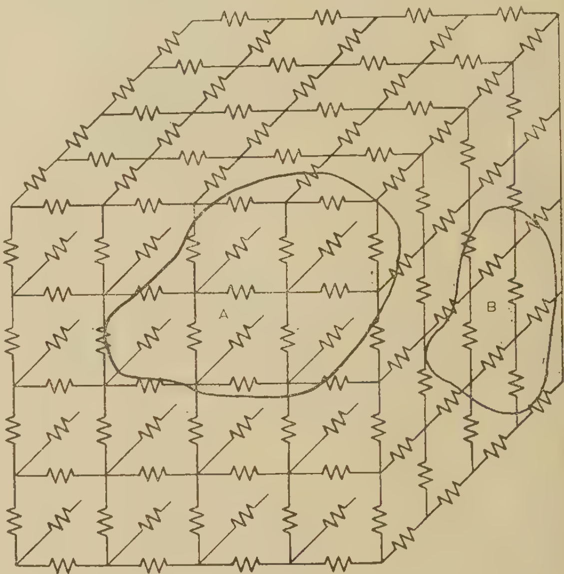


Fig. 23.—Thermal resistivities distributed through the volume of material.

equal rates of heat input at each node of the lattice, so that if the actual distribution of heat input is uniform this corresponds to a uniform distribution of the nodes; but cases of non-uniform distribution of heat input can be included by supposing a corresponding non-uniform distribution. There must also be, in the system, dissipative regions or surfaces, and these may be considered in the model as some regions maintained at a fixed temperature taken as zero.

The model is a lumped linear network with equal inputs of heat to each of the $n_a$ nodes within a volume $A$, to each of the $n_b$ nodes within a volume $B$, and so on. It is then easy to show that the mean temperatures of the nodes over $A$, say $\theta_{ma}$, the mean temperatures of the nodes over $B$, say $\theta_{mb}$, and so on, can be identified with those of the nodes of a simple network with one node only corresponding to each part $A$, $B$ . . ., etc., and the heat inputs at the nodes corresponding to the total inputs to the respective parts.

The equivalence may be established by using matrix notation, but this is somewhat cumbersome, and the following argument is simpler.

Consider on the one hand the fine mesh, and on the other hand a simple network with nodes $A$, $B$, etc., with heat inputs equal to the total inputs to the respective parts. Suppose there are $N$ such active parts. The number of conductances in a network of $N$ parts, one between each pair and one from each part to the ambient medium, is $N(N-1)/2 + N = (N^2 + N)/2$. As a procedure to determine the equivalent network, suppose that the experiment was made on the actual structure of applying unit heat input to each part in turn, and measuring the mean temperature rises that resulted. The conductances of the simple network are then to be given values such that the same ter

perature rises are produced. As there would be $N^2$ observed temperatures and corresponding equations and only $(N^2 + N)/2$ conductances to be determined, it might appear that the equations would be redundant. If, however, the relationship of reciprocity applies, then of the $N^2$ temperatures, $(N^2 - N)/2$ pairs are necessarily equal as shown by tabulating

$$\frac{N^2 - N}{2}\text{ entries}\left\{\begin{matrix} \theta_{11} & \theta_{12} & \theta_{13} \ldots \theta_{1n} \\ \theta_{21} & \theta_{22} & \theta_{23} \ldots \theta_{2n} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \theta_{n1} & \theta_{n2} & \theta_{n3} \ldots \theta_{nn} \end{matrix}\right\}\frac{N^2 + N}{2}\text{ entries}$$

where reciprocity implies $\theta_{nm} = \theta_{mn}$. If the equations for the remaining $(N^2 + N)/2$ temperature rises were used to determine an equivalent network, this network, since it has the reciprocal property, would necessarily reproduce also the remaining $(N^2 - N)/2$ temperatures correctly. Further, both networks are linear, so that superposition applies and the same sets of temperatures would then be produced in both cases by any combination of losses. They are thus equivalent in the required sense.

Thus the proposition is established if it can be shown that, in Fig. 23, the relationship between total heat inputs to the active regions and their mean temperature rises has the property of reciprocity, i.e. that the mean temperature rise over volume $A$ per watt distributed over $B$ is the same as the mean temperature rise over $B$ per watt distributed over $A$.

This follows from the reciprocal property between the nodes of the fine mesh. For an input $\delta W_a$ at node 1 in volume $A$, let the temperatures produced at nodes 1, 2, 3 . . . be $\delta\theta_{11}, \delta\theta_{12} \ldots \delta\theta_{1n}$. For successive inputs $\delta W_a$ at each of the $n_a$ nodes of $A$, the temperatures over $B$ may be tabulated as

$$\begin{array}{lllll} & \delta\theta_{11} & \delta\theta_{12} & \delta\theta_{13} \ldots \delta\theta_{1n_b} \\ & \delta\theta_{21} & \delta\theta_{22} & \delta\theta_{23} \ldots \delta\theta_{2n_b} \\ & \cdot & & \\ & \cdot & & \\ & \delta\theta_{n_a1} & \delta\theta_{n_a2} & \delta\theta_{n_a3} \ldots \delta\theta_{n_anb} \\ \hline \text{Sums} & \theta_{b1} & \theta_{b2} & \theta_{b3} \quad \ldots \theta_{bn_b} \end{array}$$

The sum of columns $\theta_{b1}, \theta_{b2} \ldots$ gives the temperature distribution over $B$ due to $\delta W_a$ input at all the nodes of $A$. The mean temperature over $B$ is the mean of these, or alternatively the sum of all the entries in the table divided by $n_b$, or

$$\theta_{bm} = \frac{\Sigma\delta\theta}{n_b}$$

If now the reverse situation is considered, the same input per node being applied over $B$, the reciprocity of the fine mesh ensures that exactly the same set of entries again appears in the tabulation and

$$\theta_{am} = \frac{\Sigma\delta\theta}{n_a}$$

But the total inputs in the respective cases are $n_a\delta W$ and $n_b\delta W$ so that the mean temperatures per watt are

$$\frac{\Sigma\delta\theta}{\delta W \,.\, n_a \,.\, n_b} \quad\text{and}\quad \frac{\Sigma\delta\theta}{\delta W \,.\, n_b \,.\, n_a}$$

which are identical. Thus total heat inputs and mean temperature rises have the relationship of reciprocity as between any volumes, for any distribution of resistivities if heat input is uniform in the sense of equal input to the nodes of the fine lattice, and if mean temperature rise means the mean temperature of these nodes.

If the nodes are chosen uniformly distributed over the volume, the case represented is that of a uniformly distributed heat input and the mean temperatures are the means over the volumes. If, however, the loss is not uniformly distributed there is still a valid equivalent network, provided that 'mean temperatures' are understood to be calculated by giving to each element of volume a weight corresponding to the intensity of loss that occurs in it.

This theorem is important in showing that an equivalent network exists in a wide range of situations where this is not self-evident. If an apparatus has $N$ volumes over each of which the heat input, to a sufficient approximation, is uniform, then a network with $N$ nodes exists, the node temperatures of which are equal to the mean temperatures over these $N$ volumes, for all inputs.

For example, various parts of an armature winding, such as the embedded part and the two end-windings, may be regarded as separate parts for the purpose of representation by an equivalent network, although the conductance between them is wholly distributed. As another example, the density of losses in the core and teeth of an armature may be very different, but in that case the mean temperature rises over each may be approximately represented, however the internal resistivity is distributed, by treating these as separate parts.

### (14.2) A Useful Relation between Temperature Rises and Losses

The equations for heat balance in each of the $n$ parts are

$$\begin{aligned} W_1 &= K_{11}\theta_1 + K_{12}(\theta_1 - \theta_2) &&+ \ldots K_{1n}(\theta_1 - \theta_n) \\ W_2 &= K_{21}(\theta_2 - \theta_1) + K_{22}\theta_2 &&+ \ldots K_{2n}(\theta_2 - \theta_n) \\ &\cdot && \\ &\cdot && \\ &\cdot && \\ W_n &= K_{n1}(\theta_n - \theta_1) + K_{n2}(\theta_n - \theta_2) &&+ \ldots K_{nn}\theta_n \end{aligned} \qquad (12)$$

or, on rearranging,

$$\begin{aligned} W_1 &= (K_{11} + K_{12} + \ldots K_{1n})\theta_1 - K_{12}\theta_2 \ldots && -K_{1n}\theta_n \\ W_2 &= \quad -K_{21}\theta_1 + (K_{21} + K_{22} + \ldots K_{2n})\theta_2 \ldots && -K_{2n}\theta_n \\ &\cdot && \\ &\cdot && \\ &\cdot && \\ W_n &= \quad -K_{n1}\theta_1 \quad - K_{n2}\theta_2 \ldots + (K_{n1} + K_{n2} + \ldots K_{nn})\theta_n && \end{aligned} \qquad (13)$$

The solution, say for $\theta_1$, is

$$\theta_1 = \frac{\begin{vmatrix} W_1 & -K_{12} & \ldots -K_{1n} \\ W_2 & (K_{21} + K_{22} + \ldots K_{2n}) & \ldots -K_{2n} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ W_n & -K_{n2} & \ldots (K_{n1} + K_{n2} + \ldots K_{nn}) \end{vmatrix}}{\begin{vmatrix} (K_{11} + K_{12} + \ldots K_{1n}) & -K_{12} \ldots & -K_{1n} \\ -K_{21} & (K_{21} + K_{22} + \ldots K_{2n}) \ldots & -K_{2n} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ -K_{n1} & -K_{n2} \ldots (K_{n1} + K_{n2} + \ldots K_{nn}) & \end{vmatrix}} \qquad (14)$$

For the denominator in the expression for $\theta_1$, by adding all columns but the first to the first, one may substitute

$$\begin{vmatrix} K_{11} & -K_{12} & \cdots & -K_{1n} \\ K_{22} & (K_{21}+K_{22}+\ldots K_{2n}) \ldots & & -K_{2n} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ K_{nn} & -K_{n2} & & \ldots(K_{n1}+K_{n2}+\ldots K_{nn}) \end{vmatrix}$$

so that

$$\theta_1 = \frac{W_1A_{11} + W_2A_{21} + \ \ldots W_nA_{n1}}{K_{11}A_{11} + K_{22}A_{21} + \ \ldots K_{nn}A_{n1}} \quad . \quad . \quad . \quad (15)$$

where $A_{11}$, $A_{21}$ . . ., etc., are the appropriate co-factors.

$$\text{or} \qquad \theta_1 = \frac{W_1 + W_2B_{21} + \ \ldots W_nB_{n1}}{K_{11} + K_{22}B_{21} + \ \ldots K_{nn}B_{n1}} \quad . \quad . \quad . \quad . \quad (16)$$

where $B_{21} = A_{21}/A_{11}, \ldots B_{n1} = A_{n1}/A_{11}$ are ratios of the co-factors. These coefficients are necessarily the same in numerator and denominator.

The same result may be deduced in a less formal but perhaps more informative manner by postulating the property of reciprocity.

For heat input to part 1 only, the temperature rise of part 1 will be greatest, and the temperature rises of the other parts will be fractions of this, say $B'_{21}$, $B'_{31}$.

Heat balance requires

$$W_1 = K_{11}\theta_{11} + K_{22}(B'_{21}\theta_{11}) + K_{33}(B'_{31}\theta_{11}) + \ldots \quad . \quad . \quad (17)$$

$$\text{or} \qquad \theta_{11} = \frac{W_1}{K_{11} + K_{22}B'_{21} + K_{33}B'_{31} + \ldots} \quad . \quad . \quad (18)$$

Now, for loss in part 2, by the principle of reciprocity, the temperature rise of part 1 is the same per watt of loss as was the rise of part 2 for loss in part 1;

$$\text{i.e.} \qquad \theta_{12} = \frac{B'_{21}W_2}{K_{11} + K_{22}B'_{21} + K_{33}B'_{31} + \ldots} \quad . \quad . \quad . \quad (19)$$

So that, by the principle of superposition, when losses $W_1$, $W_2$ . . . occur simultaneously

$$\begin{aligned}\theta_1 &= \theta_{11} + \theta_{12} + \theta_{13} \ldots \\ &= \frac{W_1 + W_2B'_{21} + W_3B'_{31} + \ldots}{K_{11} + K_{22}B'_{21} + K_{33}B'_{31} + \ldots} \quad . \quad . \quad (2\end{aligned}$$

which is the same as the expression already given, on identifyi[ng] the primed values $B'_{21}$, etc., with the unprimed values. Thus t[he] coefficients $B$ are transfer coefficients. If loss occurs in one pa[rt] only, the values of the coefficients $B$ relating to the various oth[er] parts are the ratios in which the other parts are respective[ly] cooler than that in which the loss occurs.

**(14.3) A Product of the Means of Two Variables compared wi[th] the Average of the Instantaneous Products of the Variable[s]**

Consider two quantities varying sinusoidally, about me[an] values, but out of phase;

$$\text{i.e.} \qquad \begin{aligned} K &= \bar{K} + \hat{K}\sin(\omega t + \phi_k) \\ \theta &= \bar{\theta} + \hat{\theta}\sin(\omega t + \phi_\theta) \end{aligned} \quad . \quad . \quad . \quad . \quad . \quad (2$$

The average value of the product of these two variables ov[er] one cycle of duration $T = 2\pi/\omega$ is

$$\frac{1}{T}\int_0^T K\theta dt = \tfrac{1}{2}\hat{K}\hat{\theta}\cos(\phi_k - \phi_\theta) + \bar{K}\bar{\theta} \quad . \quad . \quad . \quad . \quad (2$$

and the difference between this product and the product of t[he] means, as a ratio to the product of the means, is

$$\frac{1}{2}\frac{\hat{K}\hat{\theta}}{\bar{K}\bar{\theta}}\cos(\phi_k - \phi_\theta)$$

The greatest degree of correlation of the variation of $K$ and [$\theta$] is when $\phi_k - \phi_\theta = 0$; the ratio then becomes

$$\frac{1}{2}\frac{\hat{K}\hat{\theta}}{\bar{K}\bar{\theta}}$$

Clearly, if the variations of $K$ and $\theta$ about the mean are sma[ll] this ratio will be very small (e.g. if $\hat{K}/\bar{K} = 0{\cdot}2$, $\hat{\theta}/\bar{\theta} = 0{\cdot}2$, [the] value is 0·02). The degree of correlation between conductan[ce] and temperature variations in practice will be far less than [in] the case of in-phase variation.

[The discussion on the above paper will be found on page 504.]

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# TEMPERATURE RISES IN ELECTRICAL MACHINES ON VARIABLE LOAD AND WITH VARIABLE SPEED

By Professor A. TUSTIN, M.Sc., Member, and J. J. BATES, Ph.D., Graduate.

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SUMMARY

In selecting an electrical machine it is required to estimate how hot the windings will become when the most onerous duty anticipated is performed. This has been difficult when the duty involved varying loads and speeds. Relationships are given by which such estimates may be based on the results of temperature measurements on a machine of the type in question resulting from sustained operation of the on-off type. The data required to specify the capabilities of a machine can be given by two curves, and so may be included on the data sheet for a machine of given type. No measurement or specification of the energy losses as such is required, the method being essentially a comparison between the set or combination of various operating conditions that constitute the duty, weighted according to the time each condition holds, with measurements for representative operating conditions on the test-bed.

Methods of carrying out suitable type-tests are discussed, and the proposals are illustrated by the results of measurements and calculations on a typical 25 h.p. direct-current mill-type motor, operating both with forced ventilation and totally enclosed.

## (1) NEED FOR IMPROVED MEANS FOR ESTIMATING TEMPERATURE RISES IN MACHINES RESULTING FROM OPERATION IN SERVICE

The methods to be described result from work undertaken by the authors as a contribution to an attempt to reduce waste in the provision and operation of electric motor drives, especially in steelworks, that is being made by the British Iron and Steel Research Association. A large proportion of the motors in steel-mills, as in many other fields of application, work on starting and stopping duties with variable loads and speeds.

Motors as at present installed are found to attain widely different temperatures when performing their respective most onerous duties. The majority are unnecessarily cool, and in some cases a smaller frame size could have been used. In a smaller proportion the machines run too hot, and unduly short life and loss of production through failure may result. Sometimes there are good reasons for the thermal capabilities of a motor not being fully used, as when a smaller motor cannot be found or designed that safely gives the required torque. This raises the separate question of the need for a more adequate range of high-torque motors. The paper deals only with the prediction of temperature rises. A simple, speedy and reasonably accurate method of relating a proposed industrial duty to the thermal capabilities of a proposed motor is one necessary contribution to the general achievement of economical motorization.

The information provided by manufacturers about the machines they supply is, however, usually inadequate for the prediction of temperature rise in the case of industrial drives that involve variable load and variable speed. The concentration of attention on the 'continuous rating' and on the 'one-hour rating', besides being inadequate for motor users, also tends, in the long run, to bias manufacturers towards designs that are not so economical as they might be. The one-hour rating in particular is grossly misleading. Adequate information on motor capabilities has never been supplied to users generally or as a matter of course, and economical motor application is impossible until this is remedied.

## (2) THE DIFFICULTIES OF TEMPERATURE PREDICTION FOR VARIABLE-SPEED INDUSTRIAL DRIVES

The paper has the limited purpose of showing that the maximum temperature rise may in suitable cases very simply be estimated from a sufficiently representative sample of the operating conditions through which the motor passes in performing its most onerous duty. By 'most onerous duty' is meant the sequence or combination of operations that result in and determine the maximum temperature rise. The term 'operating condition' is used to denote a combination of armature current, field current and speed (or some other combination of variables) that suffices to fix both the losses and the thermal conductances and air flow. In some cases a single variable may suffice to define the operating condition: for example, for a series-wound motor at normal line voltage either the current or the speed completely defines the operating condition. During starting, however, if with a given constant accelerating current, speed or armature voltage defines the operating condition.

The problem that is no doubt the greatest obstacle to economical motor selection is the prediction of the most onerous probable duty as measured at the motor by recording ammeter, voltmeter and speedometer. This problem is the subject of separate study.

The difficulties of relating a specification of varying 'operating conditions' to the corresponding temperature rises would also be very great if a complete and general solution were in question. The great majority of industrial operations are, however, of a kind that permits radical simplification of the problem, in that the periods of incidence of high or low loading are usually known to be short compared with the time-constants of heating of the motor, or even of the armature winding alone. Thus, once the temperature rises reach approximate equilibrium between heat input and heat dissipation on the average, the fluctuations of temperature from a steady value due to individual peaks of loading are small, and easily estimated if required.

The paper refers principally to cases in which the most onerous duty is an intermittent and variable schedule, but the duty may be supposed to be sustained at this most onerous intensity over periods that are long compared with the time-constant of the machine. Under these conditions the mean temperatures corresponding to an indefinitely prolonged period of the most onerous duty will be reached.

## (3) THE NETWORK OF THERMAL CONDUCTANCES IN AN ELECTRICAL MACHINE

No approach to sufficiently accurate estimation of the temperature rises in a machine is possible without recognition of its

Prof. Tustin, formerly at Birmingham University, is Professor of Electrical Engineering at the Imperial College of Science and Technology, University of London.
Dr Bates, formerly at Birmingham University, is at the Military College of Science.

structure as an assembly of such parts as stator and rotor, each with cores, windings and other parts, the distribution of whose heat inputs is distinctively related to the operating conditions.

An accompanying paper[9] has shown under what conditions such a structure may usefully be represented by a linear network of thermal conductances and capacitances, even when the thermal resistances are not localized in layers of insulation between active parts. It is shown that the steady-state temperature rises $\theta_1$, $\theta_2 \ldots \theta_n$ that result from heat inputs $W_1, W_2 \ldots W_n$ to parts $1, 2 \ldots n$ in any system of $n$ parts, assuming constant coefficients of conductance, may then be expressed in the form

$$\theta_1 = \frac{W_1 + B_{21}W_2 + \ldots + B_{n1}W_n}{K_{11} + B_{21}K_{22} + \ldots + B_{n1}K_{nn}} \quad . \quad . \quad . \quad (1)$$

$$= \frac{\text{Effective loss}}{\text{Effective conductance}}$$

with analogous expressions for the temperature rises of the other parts.

In this expression $K_{11}, K_{22} \ldots K_{nn}$ are the thermal conductances from parts $1, 2 \ldots n$ to the cooling medium or to the surroundings, or to any part assumed to be at a constant temperature and to be the part from which temperature rises are measured.

If the system is such that an analogous network of $n$ parts or nodes exists, then the coefficients $B$ in the numerator and denominator are identical. This condition may not hold exactly where there is transport of heat by the flow of the cooling medium. In this case the coefficients $B$ are not the same in numerator and denominator, but the equations are still linear, and the temperature rises may in any case be expressed by

$$\theta_1 = \frac{1}{K_1}(W_1 + B_{21}W_2 + \ldots + B_{n1}W_e) \ldots \quad . \quad (2)$$

Only this more general form will be assumed in the present paper, so that the results deduced are applicable to ventilated machines without neglect of the effects of flow of the cooling medium.

## (4) THE ADEQUACY OF THE MEAN EQUILIBRIUM TEMPERATURE RISES AS APPROXIMATING TO THE MAXIMUM TEMPERATURE RISES ON INTERMITTENT INDUSTRIAL DUTY

When a machine is heated by losses that vary with the momentary operating condition, the temperature rises of the various parts increase in an irregular manner, but as the duty is continued they settle down to a condition of fluctuation about mean values, for which the average dissipation from each part is equal to the mean loss in it. These mean temperature rises are related to the mean losses by equations of the form (1) or (2), with corresponding equations for the other parts. Since these relationships are derived from the equations for heat balance for each separate part, they apply to mean values over any period for which the changes in heat storage are small compared with the dissipations. The fluctuations of the temperatures about these mean values are due to the difference between the momentary rates of heat input and the mean rates, in conjunction with the thermal capacitances of the parts and the thermal conductances between and from them.

These fluctuations of temperature about the steady or ultimate means are very small unless the duration of periods of excess load are unusually long. This may be seen immediately by considering the maximum rate at which copper can heat up when it carries current with current densities such as are usual in machine windings during starting and accelerating.

For example, at 500 amp/cm², with the resistivity of copper (warm) of $2 \times 10^{-6}$ ohm-cm, the heat input is $0 \cdot 5$ watt/cm³. But the thermal capacitance of copper is $3 \cdot 4$ watts/cm³ per deg C. The rate of temperature rise, even if there were no dissipation, could therefore not exceed $(0 \cdot 5/3 \cdot 4)$°C/sec or about 9°C/min. The times for acceleration of industrial loads, to which the periods of high loss in the windings correspond, are almost never as long as a minute. An electric train, for example, accelerates at about 2 m.p.h. per second, and the current falls o[ff] as soon as all the starting resistance is cut out, say for example at 12 m.p.h., and this is reached in about 6 sec. An acceleration with the above current density, could never under these conditions produce more than 1°C change of temperature, or $0 \cdot 5$° above the mean. For all cases where individual high-load periods do not exceed this order of duration, the fluctuations of temperature about the mean due to them are negligible, and the mean temperatures calculated from the mean losses on the most onerous sustained working schedule are substantially the maximum temperatures that will be reached.

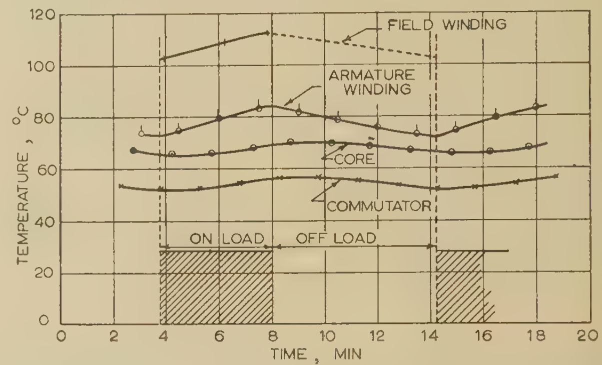


**Fig. 1.**—Temperature variations of the parts of a forced-ventilated 25 h.p. 230-volt d.c. mill-type motor with an intermittent curre[nt] of 143 amp.

Fig. 1 gives the temperature variations of the parts of a force[d-] ventilated 25 h.p. mill-type motor after the mean temperatu[re] had become steady, on intermittent loading in which the on-loa[d] periods were $4 \cdot 2$ min and the loading cycle time $10 \cdot 4$ mi[n]. Even for these comparatively long periods the fluctuations [of] winding temperature are moderate. The measurements were [by] resistance-type temperature detectors. For a totally enclose[d] motor at loads reduced correspondingly with its lower ratin[g] they are much smaller. In any case, it is very easy to estima[te] the maximum fluctuation of temperature about the mean, an[d] to make a suitable allowance. In the great majority of case[s] it is obvious that such correction would be negligible and nee[d] not be considered. Appendix 15.1 explains how the maximu[m] fluctuation would be assessed.

## (5) THE CASE OF VARYING LOSSES, BUT CONSTANT THERMAL CONDUCTANCES

### The Concept of Potential Temperature Rise

The case will first be considered, at this stage hypothetical [and] conceptual, of a machine in which the losses correspond to t[he] losses of a particular 'operating condition', and in which t[he] conductances are constant and are those for a particular spe[ed,] the 'standard speed'. The standard speed is arbitrarily chose[n]. The speed actually associated with the operating condition w[ill] usually be some other speed, but as a first step this is disregarde[d].

The steady temperature rise that would ultimately be reach[ed] by a specified part (most usefully the armature winding), if t[he] losses were constant at the values for a specified operati[ng] condition but the conductances were those of the standa[rd]

speed, will be called the 'uncorrected potential temperature rise' of that operating condition. A curve, or curves, giving these temperature rises as a function of suitable parameters defining the operating condition will be called the 'potential temperature characteristic'.

The important relationship will now be established that, for any structure for which the thermal relations are linear, in the sense that superposition applies, the mean equilibrium temperature rise due to a sequence of losses corresponding to varying operating conditions, the conductances being those of the standard speed, is simply the time mean of the corresponding potential temperature rises. This may be proved as follows.

Let the time be divided into periods $\delta_a, \delta_b, \delta_c \ldots$, short enough for the operating condition to be considered constant during each interval. The losses corresponding to the successive operating conditions during successive periods will be distinguished by the same suffixes $a, b, c \ldots$ Then the potential temperature rises $\theta_p$ of the successive operating conditions are, by definition, by eqn. (2), and with effective conductance $K_1 = K_{11} + B_{21}K_{22} + \ldots$ assumed constant,

$$\begin{aligned} \theta_{pa} &= \frac{1}{K_1}(W_{1a} + B_{21}W_{2a} + B_{31}W_{3a} + \ldots) \\ \theta_{pb} &= \frac{1}{K_1}(W_{1b} + B_{21}W_{2b} + B_{31}W_{3b} + \ldots) \qquad (3) \\ &\vdots \\ \theta_{pn} &= \frac{1}{K_1}(W_{in} + B_{21}W_{2n} + B_{31}W_{3n} + \ldots) \end{aligned}$$

The mean of the potential rises over time, $\theta_{pm}$, is obtained by multiplying $\theta_{pa}$ by $\delta_a$, $\theta_{pb}$ by $\delta_b$, etc., adding and dividing by the total time $T$. Performing these operations on both sides gives

$$\theta_{pm} = \frac{1}{K_1}\left[\frac{W_{1a}\delta_a + W_{1b}\delta_b + \ldots}{T} + \frac{B_{21}(W_{2a}\delta_a + W_{2b}\delta_b + \ldots)}{T} + \ldots\right] \qquad (4)$$

But $\dfrac{W_{1a}\delta_a + W_{1b}\delta_b + \ldots}{T}$ is the mean loss in part 1, $W_{1m}$, and similarly the second fraction involves the mean loss in part 2, $W_{2m}$, and so on.

So that

$$\theta_{pm} = \frac{W_{1m} + B_{21}W_{2m} + B_{31}W_{3m} + \ldots}{K_1} \qquad (5)$$

and this, it has already been shown, is the mean temperature rise that would be reached on account of the fluctuating losses, but with the conductances those of the standard speed. This relation gives the mean temperature rise directly in cases such as transformers, in which conductances are constant, or such as rotating machines with constant speed provided this is the standard speed.

## (6) THE CASE WHEN THE SPEED AND THE THERMAL CONDUCTANCES ARE ALSO VARIABLE

### (6.1) General Effect of Speed

In most cases the operation of machines also involves variation of speed, and it is required to take this into account. The effect of speed will be considered in several steps. First, the case will be considered in which the conductances are constant, but have values corresponding to some speed other than the standard speed. It will be shown that this can be taken into account by a simple factor correcting for speed. Then the case of varying conductances, corresponding with varying speed, will be considered, and it will be shown that the mean temperature is not significantly affected by the variation of the conductances about their mean values but depends only on the mean values. Thirdly, it is pointed out that the mean values of the conductances are likely to be practically identical with the conductances at the mean speed, provided appreciable periods of standstill are not involved, for at standstill or very low speeds there is likely to be a marked departure from linearity of the conductance/speed relationship. Finally, it is shown how the effect of such non-linearity can be taken into account.

### (6.2) Conductances Constant at Values corresponding to a Speed other than Standard Speed

The mean temperature rise with the conductances those at the standard speed is given by an expression of the form

$$\theta_{pm} = \frac{1}{K_1}(W_{1m} + B_{21}W_{2m} + B_{31}W_{3m} \ldots)$$

the part being designated by the suffix 1; and when the conductances are those of some other speed, both $K_1$ and $B_{21}$, $B_{31} \ldots$ are different on that account. Let their values be $K'_1, B'_{21}, B'_{31} \ldots$ and let the mean temperature rises be $\theta'_{pm}$. Then, the mean temperature rise is changed in the ratio

$$\frac{\theta'_{pm}}{\theta_{pm}} = \frac{K_1}{K'_1}\,\frac{W_{1m} + B'_{21}W_{2m} + B'_{31}W_{3m} + \ldots}{W_{1m} + B_{21}W_{2m} + B_{31}W_{3m} + \ldots} \qquad (6)$$

The quantity most affected by speed is $K_1$, which is a weighted sum of the conductances from all the parts. The factors $B$ change less, since they also involve conductances between the parts. The factor $K_1/K'_1$ is a function of speed only, but the remaining factor is a function of speed that differs somewhat according to the ratios of the various losses. If the loss were confined to part 1 this factor would not be affected by speed at all. If the loss were wholly in any other part, this factor would be a function of speed only. If the losses in the various parts were in all cases in the same ratio, this factor would again be a function of speed only; for, let

$\dfrac{W_{2m}}{W_{1m}} = \alpha_2$, $\dfrac{W_{3m}}{W_{1m}} = \alpha_3$, etc.; then the factor is

$$\frac{1 + B'_{21}\alpha_2 + B'_{31}\alpha_3 \ldots}{1 + B_{21}\alpha_2 + B_{31}\alpha_3 \ldots} \qquad (7)$$

which depends only on speed for $\alpha_2, \alpha_3 \ldots$ constant. For various different distributions of losses, however, this factor is a slightly different function of speed, for but the reasons given, and because $K_1/K'_1$ is the dominant factor, the ratio of temperature change is mainly a function of speed and is little affected by the ratios of the losses.

That this is so may be shown by considering the limiting cases of the variation of loss distribution, namely the hypothetical cases in which all the loss occurs in part 1, part 2, etc., in turn.

Measurements of the values of the terms $1/K_1, B_{21}/K_1, \ldots$, etc., were obtained for a range of speeds on a representative 25 h.p. mill-type motor in a manner described in a companion paper.[9] The values of the ratio of temperature change $\theta'_{pm}/\theta_{pm}$ from eqn. (6) were calculated, using these measured values, for the cases in which the loss was confined in turn to the commutator, the core and the armature winding. The factors for change of temperature rise of the winding for each case are plotted as functions of speed in Fig. 2. It will be seen that the correction is not greatly affected by the different loss distributions even in this extreme case.

As in practice the losses are distributed between the several

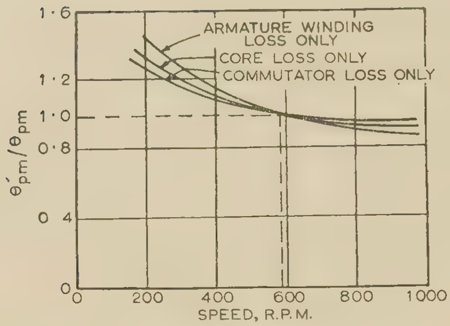


Fig. 2.—Effect of distribution of losses on change of temperature with speed.

parts, the effect of change of speed is always intermediate between the extreme cases of all the loss being in one part or all in another, so that the range of possible variation is quite small. It is considered, therefore, that over the practical range of service conditions the effect of speed may adequately be represented by a single curve, taken to apply irrespective of the actual ratio of losses, it being understood that this curve will be obtained by measurements under conditions in which the loss distribution is fairly representative of the distributions of mean loss likely to occur in the service conditions to which the curve is intended to be applied. Correction of the mean potential temperature rise by a factor $\theta'_{pm}/\theta_{pm}$, which is a function of speed, therefore provides the temperature rise that would occur with the conductances those of a non-standard speed.

### (6.3) Effect of Variation of Conductances about their Mean Values

Transfers of heat are proportional to the product of temperature differences and conductances, so that if the temperature differences were constant the mean rates of heat transfer would be the same as if the conductances were constant at their mean values. In operation in service, both the conductances and the temperature difference vary. If periods of larger temperature difference regularly tended to coincide with those of greater conductance, and periods of smaller temperature difference with those of smaller conductance, the mean rate of heat transfer would be somewhat greater than the product of mean temperature difference and mean conductance. If the variations tended to be in opposite directions the heat transfer would be smaller. It is shown, however, in the companion paper[9] that this effect is negligible in the conditions under consideration, mainly because the temperature fluctuations are small. The effect is reduced still further because changes in temperature differences are not in phase with the changes of conductance.

### (6.4) Correction for Speed when Mean Conductances do not correspond to those at Mean Speed

In a rotating machine the conductances that vary with speed are those from the surfaces to the ventilating air. If these conductances are related linearly to speed over the range of speeds in question, the mean conductances are equal to those at the mean speed. On this assumption the simple final result is that the mean temperature rise in service is the mean potential temperature rise corrected for the ratio of mean speed to standard speed, independently of how a particular mean speed is constituted by different speeds at various times.

The assumption of approximate linear dependence of conductance on speed is justified by consideration of published data and by measurements by the authors, over a considerable range of speed variation, but these measurements also showed that near standstill the conductances may, for various reasons, depart considerably from the linear variation at higher speeds.

In self-ventilated machines it would be expected that the transfer of heat from the rotor to the ventilating air would fall rapidly at speeds very near to zero, because of the cessation of replacement of air in the air-gap or ducts, this air becoming relatively static and warming up.

In forced-ventilated machines one would not expect any such effect. On the 25 h.p. direct-current forced-ventilated motor that was investigated an opposite effect occurred at low speeds, the conductances increasing. This appears to be a peculiarity of the particular design and has been dealt with fully in the companion paper.[9]

One procedure that avoids error due to possible anomalous departures of the conductances from linearity at or in the neighbourhood of standstill is to calculate an effective mean speed in service by counting standstill periods not as zero speed but as some suitable small speed, positive or negative, such as corresponds to an extension of the linear relation between conductance and speed over the rest of the range.

Another procedure is to correct the temperature rise directly for departures of the mean conductance in service from some standard conductance. To do this requires the variation of conductance with speed to be known for the motor under consideration, so that the mean conductance can be found from the schedule of speeds in the service. A method of test for finding the effective conductance as a function of speed is described in the companion paper.[9]

These procedures will be discussed further in connection with the method proposed to obtain temperature characteristics by testing.

## (7) OUTLINE OF PROPOSED BASIS FOR TEMPERATURE ESTIMATES

### (7.1) Data to be obtained by Testing

A simple basis thus exists for the estimation of the maximum temperature rises that can be produced by indefinitely prolonged continuation of operation on variable load and variable speed on any schedule, an adequate sample of which is specified. All that need be known about the duty is the fraction of the total time spent at various operating conditions.

The capabilities of the machine may be expressed by two curves giving respectively

(*a*) Values of uncorrected potential temperature rise for a sufficient range of operating conditions and for a stated standard speed.

(*b*) Correction factors for temperature rise, for use when the mean operating speed differs from the standard speed.

These two curves are called respectively the 'temperature characteristic' and 'cooling correction' curves. Figs. 3 and 4 show examples of such curves for the 25 h.p. motor, forced-ventilated, obtained by tests in a manner described below.

The temperature rise for the service is obtained by reading the potential temperature rises from the curve, for the operating conditions that occur, averaging over time, and multiplying by a correction factor corresponding to the mean speed; or, if there is anomalous cooling at standstill, corresponding to an 'effective' mean speed in which standstill has been given an appropriate non-zero equivalent speed.

### (7.2) Methods of obtaining, by Test-Bed Measurements, Temperature Characteristic and Cooling Correction Curves

Conditions may be produced on the test-bed in which the potential temperature rises (or, more precisely, definite fractions

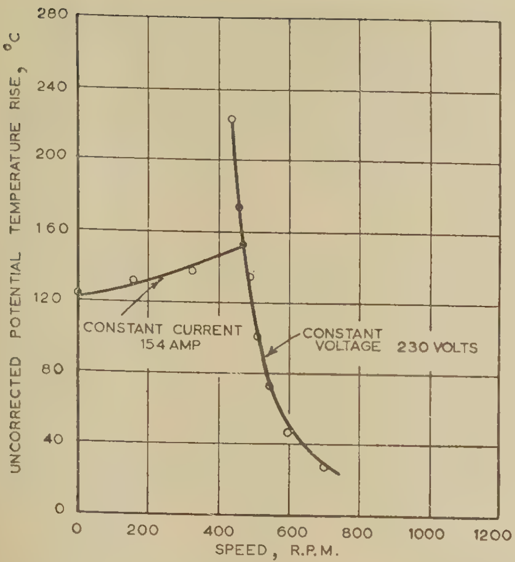


**Fig. 3.**—Temperature characteristic curve for a 25 h.p. mill-type motor, forced ventilated.

Standard speed, 590 r.p.m.
Standard temperature, 90° C.

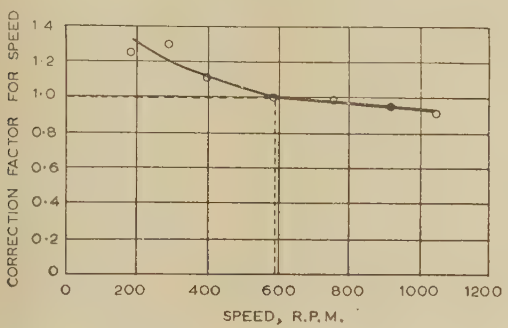


**Fig. 4.**—Cooling correction curve for a 25 h.p. mill-type motor, forced ventilated.

Standard speed, 590 r.p.m.

of them) become observable or measurable, using a form of on-off loading. The machine is run alternately for a period of a few minutes at the operating condition in question, and then for a few minutes at a selected speed but with approximately zero losses, as indicated by Fig. 5. If the ratio of 'on' time to total time is the fraction $a$, the mean losses are $a$ times those of the operating condition. If the successive speeds are $n_1$ and $n_2$, the mean speed is $an_1 + (1 - a)n_2$, and $n_2$ may be chosen so that this is equal to the standard speed. The temperature rises are

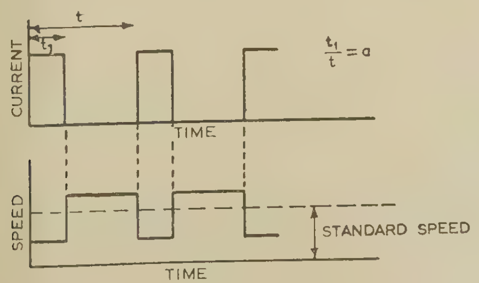


**Fig. 5.**—Type of on-off loading required for the data runs.

then the fraction $a$ of the potential temperature rises. This applies to any one of the various parts, but in practice it is most usefully applied to the windings, since their temperatures are usually the limiting feature in service.

This leaves $a$, the fraction of time 'on', open to choice, but it is desirable to choose this ratio so that the temperature rise $(a\theta_p)$ actually reached is approximately the value regarded as the normal limit for service operation, or gives a particular temperature related to this value, which will be referred to as the 'standard temperature'. This precaution avoids possible damage by excessive temperatures during testing, but it also has the important advantage of avoiding or minimizing the need for correction on account of the variation of losses in the windings with temperature.

If the standard temperature is attained on the test, then the $i^2r$ losses, when steady conditions have been reached, will be those that occur with the winding at the standard temperature, and a calculation of a temperature rise in service, based on these data, will implicitly assume the $i^2r$ losses to be those that occur if the windings are at this standard temperature. This is always safe if used to establish that the temperature rise in service will not exceed this standard value. If the calculation showed some lower temperature, the temperature actually to be expected would be somewhat lower still, because the actual mean $i^2r$ losses would be less than implicitly assumed.

Probably, however, even if the ratio of 'on' time to total time is selected so that the temperature rises observed are about right, they will not be exactly right. An expression by which an approximate correction may be made for the discrepancy in $i^2r$ losses is developed in Appendix 15.2. It requires that one should know, or guess, the proportion of the observed temperature rise due to $i^2r$ loss as distinct from the losses that are not temperature dependent. The same relationship may be used to correct a temperature estimate for a given duty if the first estimate differs from the standard temperature rise.

## (8) PROBABLE ACCURACY AND RANGE OF APPLICABILITY OF THE PROPOSED METHODS

It is desirable, in proposing a method of estimation that involves certain assumptions and approximations, to produce experimental evidence of its accuracy and of its possible errors. In the case of the present proposals there are difficulties in doing this directly.

No possibility exists of useful exploration of the relationship of possible errors to types of service by comparison of estimated temperatures with those measured under service conditions in various services. First, the operating conditions in service are more liable to variation than are the relationships to be tested. Secondly, even if the first difficulty were removed by continuous recording of currents, etc., in service over long periods, there would be no obvious assurance that, because temperatures on one or more work schedules were correctly predicted, all others would be predictable with the same accuracy. Thirdly, the method does not purport to take account of all peculiarities of operation in service. It gives a basis for judgment by predicting the temperature rise that would be reached under conditions on the test-bed at specified variable loads and speeds. If, for example, a totally-enclosed crane motor has additional cooling in service due to the motion of the crane, this must be allowed for separately unless the basic tests on the test-bed from which data are drawn were made with a similar draught over the frame. Any comparison with measurements in service would be as likely to reflect such factors as to reflect errors in the relationships proposed.

A second possibility would be to compare calculations with

measurements made by simulating 'run curves' on the test-bed, using flywheel loads or the like. This, apart from the difficulty of equipment, is open to much the same objections. Such procedures have been thought unpromising and have not been attempted.

A much more useful insight into the scope for safe application is obtainable indirectly, from the consideration that what is really involved is a comparison between two sets of tests: the on-off tests whose results are recorded in the data curves, which will be called 'data runs'; and the hypothetical operations under the same test-bed conditions at the varying loads and speeds of the duty, which will be called 'service runs'. The use of interpolation, in drawing the curves, may be thought of as providing, without error, the results that would have been given by an indefinitely finely graded sequence of data runs. What is in effect being done in averaging potential temperature rises is to pick out from the data runs a combination that includes the same operating conditions, for the same proportions of time as the service runs, associated with a proportion of no-load operation such that the mean speed is also the same as on the service runs. The fact that the actual durations and sequence of the individual loading periods differ between the data runs and the service runs, does not in itself involve significant difference so long as the periods are short.

Since the heat inputs involved are the same, the only assumption that needs examination is that the mean conductances over the service runs are the same as over the data runs. It has already been mentioned that non-linearity in the relationship of conductance to speed may impair this equality, and it is required to assess the amount of any such error and to consider how it may be avoided.

### (8.1) Possible Error due to Relationship between Conductance and Speed being Non-linear

If there is a similar distribution of speeds in the data runs and the service runs, any such error tends to be to some extent self-cancelling, but the spread of speeds will not usually be the same for both.

If the relationship of effective conductance to speed were known, no error need be incurred. The speeds and times in the data runs could be arranged so that the temperatures were determined at a particular mean conductance instead of a particular mean speed. Correction would then be made according to the ratio of the mean conductance in service to this 'standard' conductance, instead of according to the ratio of the mean speed in service to the standard speed. This would add to the work but eliminate possible error. Strictly, several distinct conductances are involved, and ideally the tests should be arranged so that they all have 'standard' values, but refinement of this sort is impracticable and unnecessary. It is shown in the companion paper[9] that it is possible to measure the total rate of heat dissipation from the armature winding per deg C rise (when the loss is confined to the armature winding) as a function of speed. The shape of this curve may be taken to represent the dependence of effective cooling on speed, and it may be used to select test speeds to give a standard conductance. The mean conductance in service would also be assessed from the same curve, the cooling correction being the ratio of this mean conductance to the standard conductance. It was not realized while tests were being carried out that, for the 25 h.p. motor tested, non-linearity would be so marked with the motor forced-ventilated. If this had been foreseen, the tests for the potential temperature characteristics would have been made in this way. Knowing the form of conductance/speed variation, however, it has been possible to assess the difference between the potential temperature rises obtained for a mean speed of 590 r.p.m. and those that would have been obtained at a mean conductance corresponding to the conductance at that speed. This comparison is shown as Fig. 6; the conductance/speed curve is given in Fig. 7.

There are many applications in which, even on the most onerous duty, the motor is at rest for a considerable proportion

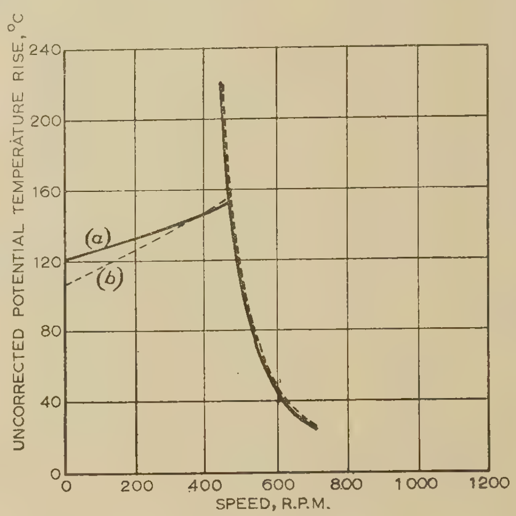


Fig. 6.—Comparison of potential temperature rises for:
(a) Tests carried out with a mean speed of 590 r.p.m.
(b) Predicted temperature if tests had been carried out with a mean conductance equal to that at 590 r.p.m.

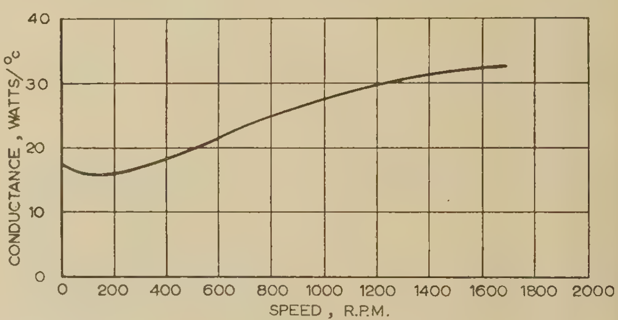


Fig. 7.—Form of the conductance/speed variation for the forced ventilated motor tested.

of the total time, so that the conductance at standstill is of special importance. The effect of the conductance at standstill differing from the value corresponding to a straight-line relation to speed can be taken into account by attributing to standstill some equivalent positive or negative speed. Fig. 8 shows the equivalent

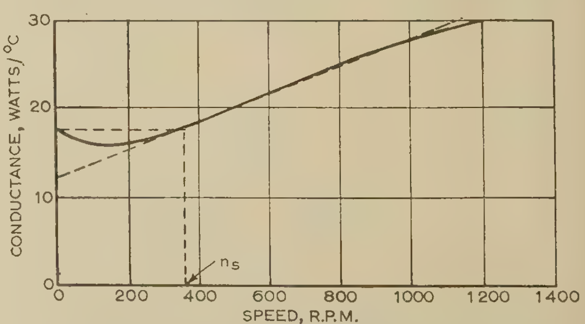


Fig. 8.—Allowance for a departure from linearity at standstill by giving to standstill periods a fictitious speed $n_s$.

speed which must be assumed in place of zero for standstill periods in the case of the particular non-linear conductance/speed relationship found in the forced-ventilated motor that was tested. To determine this speed it is not necessary to know the form of the conductance/speed curve completely. Three tests suffice: one to find the conductance at standstill and the others to find the conductances at two speeds on the linear part of the conductance/speed characteristic, say at 500 and 1 000 r.p.m. in this particular case.

### (9) DETERMINATION BY TESTS OF POTENTIAL TEMPERATURE CHARACTERISTIC AND CORRECTION FOR SPEED FOR A 25 H.P. MILL-TYPE MOTOR

A type-600 d.c. mill-type series-wound motor was coupled to a similar motor and to a small auxiliary driving motor, and connected for back-to-back testing. A booster in series with the armatures provided the voltage for circulating current. Starting and stopping were controlled by contactors from a clock-and-relay timing device which is described in detail in the companion paper.[9] Hand-operated rheostats were included for speed and current adjustment.

The motor was fitted with resistance temperature detectors and slip rings for the measurement of temperatures at various points on the armature. This provision is not necessary for carrying out the required measurements, but it enabled supplementary information to be obtained and provided a useful check on measurements of the temperature rise by resistance of the armature winding on shut-down.

The motor was subjected to sustained on-off tests, with overall periods of about 8 to 10 min, at various operating conditions during the 'on' periods and with various speeds during the 'off' periods, maintained by the auxiliary driving motor. The ratio of 'on' to 'off' time was selected in each case so that, as nearly as could be judged, and in the light of previous tests, the 'standard temperature' of 90° C would be reached by the winding. If a temperature different from the standard was reached, this was corrected for the effect of the extra $i^2r$ losses by the method of Appendix 15.2, so as to give a result for the loss corresponding to the standard temperature. As the correction is only about one-fifth of any discrepancy in the working temperature, such correction is not likely to be a source of appreciable error.

The two series of tests were made that are required to provide complete information about the thermal capabilities of a motor, namely a series to determine the potential temperature rises at a standard speed, for which 590 r.p.m. was selected, and a further series at a fixed operating condition but at various mean speeds to obtain the characteristic of correction for speed. The standard speed selected was not well chosen. A much lower speed would have been better, since investigation of work schedules subsequently showed that the average operating speed is usually much lower.

The characteristics obtained for the forced-ventilated motor have already been presented in Figs. 3 and 4. The corresponding characteristics for the same motor working totally enclosed are shown in Figs. 9 and 10. Fig. 11 shows the form of conductance speed variation for the totally-enclosed motor: there is not the same marked departure from linearity at low speeds as in the forced-ventilated case.

For both totally-enclosed and forced-ventilated conditions the potential temperature data, in addition to the curve that gives the potential temperature for any current at normal line voltage, include points, lying on distinct curves, for a particular current and speeds down to zero, which provide values of potential temperature for conditions during the starting period when starting at that current. If the starting current may have any of a range of values, several such curves will be necessary.

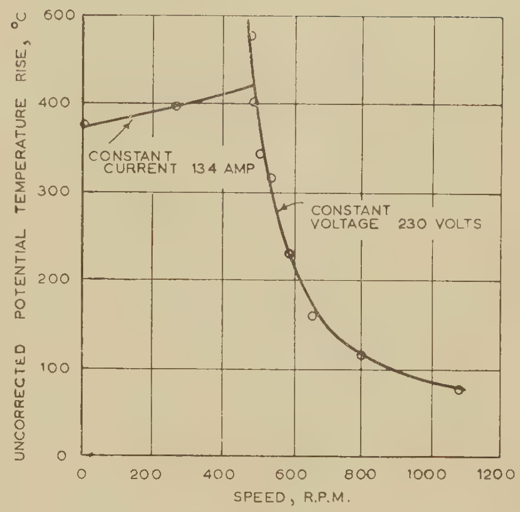


**Fig. 9.**—Temperature characteristic curve for a 25 h.p. mill-type motor, totally enclosed.

Standard speed, 590 r.p.m.

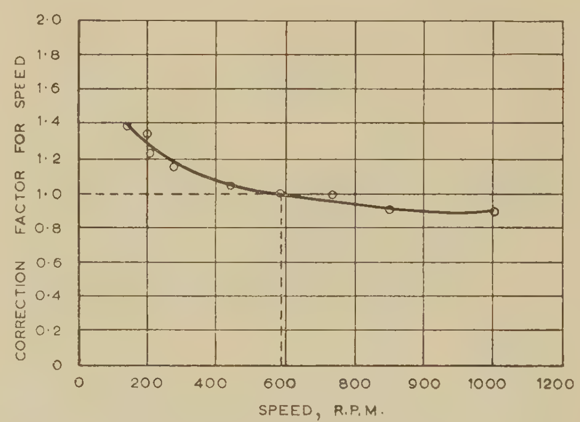


**Fig. 10.**—Cooling correction curve for a 25 h.p. mill-type motor, totally enclosed.

Standard speed, 590 r.p.m.

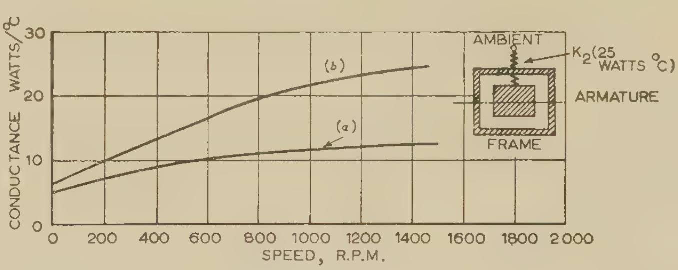


**Fig. 11.**—Form of the conductance/speed variation for the totally-enclosed motor tested:

(a) Thermal conductance from armature winding to surroundings = $K_1K_2/(K_1 + K_2)$.
(b) Thermal conductance from armature winding to frame = $K_1$.

The characteristics of potential temperature may alternatively be plotted as functions of current, since for a series motor either speed or current may be used to distinguish operating conditions. In this form, the results for the forced-ventilated and for the totally-enclosed motors appear as in Fig. 12.

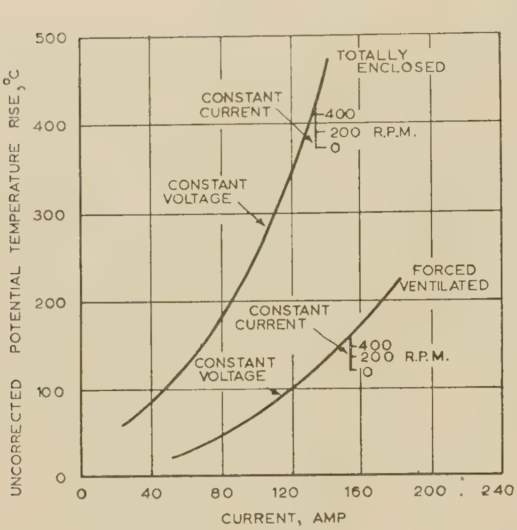


Fig. 12.—Potential temperature characteristics as a function of current instead of speed.

## (10) THE PROPOSED METHOD AS A MODIFICATION OF THE WELL-KNOWN 'ON-OFF', OR 'LOAD-FACTOR', METHOD OF RATING

It will be noted that the testing procedure required in the proposals that have been outlined does not differ essentially from that of the more familiar 'on-off', or 'load-factor', method of rating.

What is now proposed differs from the traditional manner of using on-off tests in two respects: first, in a systematic arrangement of a series of such tests in respect of the average speeds used; and secondly, in the form of presentation of the results. The testing required is of the same kind and is no more than is in any case necessary to specify adequately the thermal capabilities of the machine.

It has been usual in on-off testing to record for various loads the ratio of load-on time to total time that produces the given limiting temperature rise $\theta$. This is equivalent to the recording of the potential temperature rise, which is simply $\theta \times$ (total time)/('on' time). It is now proposed that the main set of tests should be at a constant mean speed, and that a supplementary set should be made at constant load, but with the mean speed systematically varied.

These simple changes make possible a direct deduction of the temperature rises due to any working schedule, by averaging the potential temperature rises and correcting the result for schedule speed. This is possible only when such tests have been made in a systematic manner and the results for various loads have been recorded in the form of temperature rises multiplied by (total time)/('on' time), rather than in the form of the on-off times that produce a particular temperature rise.

Where substantial 'on-off' test results exist for a machine, it may be necessary to supplement them only by a series of tests to obtain the curve for temperature corrections with speed in order to make use of the existing data to plot the curve of potential temperature rise. Data for conditions other than those of full voltage may also have to be added to cater properly for starting periods.

## (11) AN EQUATION FOR A POTENTIAL TEMPERATURE CHARACTERISTIC

The potential temperature characteristic usually required is a curve of winding temperature against speed, or current, under constant conditions for heat dissipation. The variation of the potential temperature rise with current is thus solely due to variation in the losses in the machine, and it is interesting to see how the curve is related to the manner of variation of the losses.

Equations of the form $A + BI^2$ fit the measured potential temperature characteristics for both the totally-enclosed and forced-ventilated machines extremely well, as is shown in Figs. 13 and 14. This may be interpreted as the constant $B$ taking

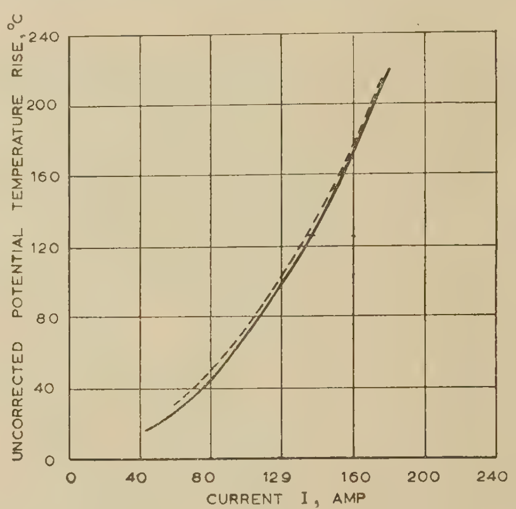


Fig. 13.—An equation for potential temperature characteristic of forced-ventilated motor.

——— Measured.
– – – – Plotted from the equation $\theta_p = 8 + 0{\cdot}0066 I^2$.

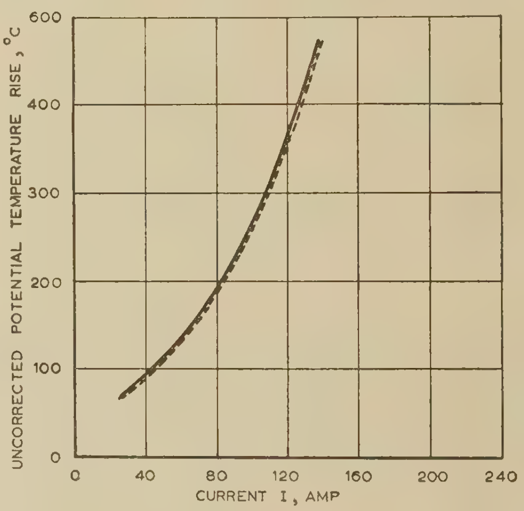


Fig. 14.—An equation for potential temperature characteristic of totally-enclosed motor.

——— Measured.
– – – – Plotted from the equation $\theta_p = 40 + 0{\cdot}021 I^2$.

account approximately of the winding loss and the constant taking account approximately of the iron and constant losses. If a relation of this form is assumed, two tests only suffice to determine the temperature characteristic. This provides the possibility of an important reduction in testing time.

## (12) CONCLUSION

It has been shown that the mean temperature rise of a specific part, e.g. the armature winding, of a motor operating on a short

run service with variable load and speed may be related to the results of systematically arranged on-off tests on the test-bed.

The possible errors have been shown to be small, provided that the thermal conductances for heat dissipation vary linearly with speed, and means of avoiding such possible errors have been indicated.

## (13) ACKNOWLEDGMENTS

The authors are grateful to the General Electric Company for the loan of machines and other practical help in the work described, and to the British Iron and Steel Research Association both for the loan of machines and for the bursary which enabled one of them to participate in the investigation.

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## (15) APPENDICES

### (15.1) Correction for Fluctuation of Temperature about the Mean

The purpose of this Appendix is to show how the maximum temperature rise may be estimated by making a correction to the mean temperature rise calculated as described. It will also appear that such correction is so small as to be unnecessary in all but exceptional cases.

Measurements showed that the variations in core temperature are much smaller than those in winding temperature, and, to a first approximation, the core temperature may be regarded as constant. The variations in winding temperature about the mean can then be calculated sufficiently accurately as the temperatures due to the excess and deficiency of the loss in the windings over or under the mean loss. The winding has a thermal capacitance $C$ and is connected to the core (regarded as a constant-temperature ambient material) by a thermal conductance $K$. This is analogous to an electrical resistance and capacitance in parallel with a varying current input, e.m.f. then corresponding to temperature.

For a loss varying in the particularly simple manner shown in Fig. 15, the maximum of the variation of the winding temperature above the mean is given as follows.

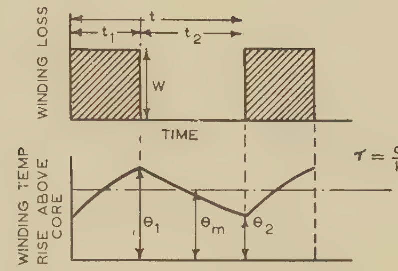


**Fig. 15.**—Simple variation of temperature about a mean due to intermittent periods of excess loss.

Referring to Fig. 15, and with the ratio $C/K$, which is the time-constant $\tau$, the temperature $\theta_1$ at the end of period $t$, is,

$$\theta_1 = \frac{W}{K}(1 - \varepsilon^{-t_1/\tau}) + \theta_2 \varepsilon^{-t_1/\tau} \quad . \quad . \quad . \quad (8)$$

and the temperature $\theta_2$ at the end of period $t_2$ is

$$\theta_2 = \theta_1 \varepsilon^{-t_2/\tau} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

from which

$$\theta_1 = \frac{W}{K}\left(\frac{1 - \varepsilon^{-t_1/\tau}}{1 - \varepsilon^{-t/\tau}}\right) \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and

$$\theta_1 - \theta_m = \frac{W}{K}\left(\frac{1 - \varepsilon^{-t_1/\tau}}{1 - \varepsilon^{-t/\tau}} - \frac{t_1}{t}\right) \quad . \quad . \quad . \quad (11)$$

When $t/\tau$ becomes small the rises and falls of temperature become very nearly linear. The slope of the rising curve is then

$$\frac{W}{K}\frac{t_2}{t}\frac{1}{\tau}$$

and that of the falling curve

$$-\frac{W}{K}\frac{t_1}{t}\frac{1}{\tau}$$

The value of $\theta_1 - \theta_m$ is then given by

$$\theta_1 - \theta_m = \frac{1}{2}\frac{W}{K}\frac{t_2}{t}\frac{t_1}{\tau}$$

$$= \frac{1}{2}\frac{W}{K\tau} t_1\left(1 - \frac{t_1}{t}\right) \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$= \frac{1}{2}\frac{W}{C} t_1\left(1 - \frac{t_1}{t}\right) \quad . \quad . \quad . \quad . \quad . \quad (13)$$

If, also, $t_1/t \ll 1$, this reduces to

$$\theta_1 - \theta_m = \frac{1}{2}\frac{H}{C} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where $H = Wt_1$, the total heat input, now entirely taken up during the period $t_1$ by the thermal capacitance.

The ratio of the quantities on the respective sides of eqns. (12) and (11) gives

$$P = \frac{\frac{1}{2}\frac{t_1}{\tau}\left(1 - \frac{t_1}{t}\right)}{\frac{1 - \varepsilon^{-t_1/\tau}}{1 - \varepsilon^{-t/\tau}} - \frac{t_1}{t}}$$

and this ratio is plotted in Fig. 16 against $t/\tau$ for particular values of $t_1/t$. The curves show that for quite large values of $t/\tau$ the simple equation (13) is quite adequate. They can also be used to

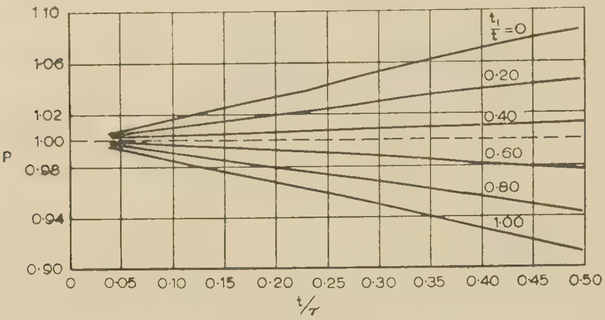


Fig. 16.—Ratio $P = \dfrac{\frac{1}{2}\frac{t_1}{\tau}\left(1 - \frac{t_1}{t_2}\right)}{\dfrac{1-\varepsilon^{-1/\tau}}{1-\varepsilon^{-t/\tau}} - \dfrac{t_1}{t}}$ for values of $t/\tau$ and $t_1/t$.

correct a temperature calculated from eqn. (13) to obtain the accurate value given by eqn. (11). These curves, together with eqn. (13), give a rapid and easy method of finding the temperature rise above the mean for any value of $t/\tau$.

### (15.2) Correction to Allow for Variation of $i^2r$ Loss with Temperature

The need for correction for variation of the resistances of windings with temperature may arise in two cases.

(*a*) The potential temperature rises represent the heating effect of the losses at the various operating conditions and with the $i^2r$ losses as for a 'standard' temperature. Ideally, the on-off time ratio could be so chosen that the mean temperature reached would be the standard temperature, but in practice it will usually be somewhat more or somewhat less. The mean temperature recorded then corresponds with losses at that temperature, i.e. the one actually reached, and it is required to find what the temperature would have been had the $i^2r$ losses been those corresponding to the standard temperature.

(*b*) In using values of the potential temperature rise (valid for standard temperature) to calculate the temperature on a specified duty, the calculation gives the temperature on the assumption that the resistances of windings are those at standard temperature. The resistances will actually be those for whatever temperature is reached in thermal equilibrium.

In both cases four distinct temperatures are involved, namely

| | |
|---|---|
| Temperature of surroundings .. .. .. .. | $\theta_a$ |
| Standard temperature .. .. .. .. .. | $\theta_s$ |
| Actual or balancing temperature rise above $\theta_a$ at which the losses (for that temperature) are dissipated .. | $\theta_b$ |
| Temperature rise (fictitious or calculated) above $\theta$ at which the losses for the standard temperature are dissipated .. .. .. .. .. .. .. | $\theta_c$ |

In case (*a*) one measures $\theta_b$ and $\theta_a$ and requires $\theta_c$. In case (*b*) one calculates $\theta_c$ and requires $\theta_b$, given $\theta_a$. Thus what is required is the relation between $\theta_s$, $\theta_b$, $\theta_c$ for any given $\theta_a$.

Let the temperature rise due to the $i^2r$ loss only, with $r$ at the standard temperature $\theta_s$, be $\theta_r$. The relationship between $\theta_b$, $\theta_c$ and $\theta_s$ might be as shown in Fig. 17, which illustrates the case for $\theta_b > \theta_c$.

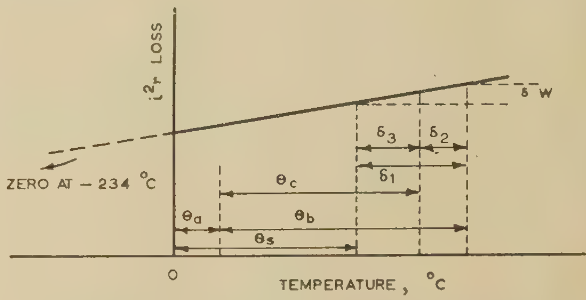


Fig. 17.—Variation of $i^2r$ loss with temperature and its relation to standard, measured and derived temperatures.

Writing
$$\delta_1 = \theta_b + \theta_a - \theta_s$$
$$\delta_2 = \theta_b - \theta_c$$

Then, since the $i^2r$ loss only ($r$ at $\theta_s$) causes a temperature rise $\theta_r$, the extra loss $W$ due to the temperature difference $\delta_1$ causes a temperature rise $\theta_r/(234 + \theta_s)$ per degree.

But this is to account for $\delta_2$, so that

$$\delta_2 = \frac{\delta_1}{234 + \theta_s}\theta_r$$

and
$$\frac{\delta_3}{\delta_1} = \frac{\delta_1 - \delta_2}{\delta_1} = 1 - \frac{\theta_r}{234 + \theta_s}$$

Thus the departures of the 'standard loss' temperature and of the actual balancing temperature respectively from the standard temperature are always in the ratio $1 - \theta_r/(234 + \theta_s)$, which is known if $\theta_r$ can be found. This relationship holds for both cases (*a*) and (*b*).

A rough estimate of $\theta_r$ will suffice, for $\theta_r/(234 + \theta_s)$ is usually of the order of $0{\cdot}2$, so that if $\theta_r$ is wrong, say about 10%, the correction is wrong by $(1 \pm 0{\cdot}22)$ replacing $(1 \pm 0{\cdot}20)$, and applied to differences that will rarely exceed 10° to 15°C the error in the final estimate is very slight. Ideally the temperature rises due to $i^2r$ loss alone may be measured, using the method indicated in the companion paper,[9] but it will usually suffice to make use of the approximately known ratio of $i^2r$ and 'other' losses to the total losses, and to assume that the 'other' losses are only about 60% as effective in heating the winding as are the $i^2r$ losses themselves. Then the fraction of the observed or calculated temperature rise attributable to $i^2r$ losses is given immediately.

[The discussion on the above paper will be found on page 504.]

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April 1956

# PERFORMANCE AND HEATING CURVES FOR MOTORS ON SHORT-RUN DUTIES

By Professor A. TUSTIN, M.Sc., Member, D. F. NETTELL, B.Sc., A.M.I.Mech.E., Associate Member, and R. SOLT, B.Sc., A.M.I.Mech.E., Associate Member.

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## SUMMARY

The paper shows how curves may be prepared that facilitate the design or selection of motors for services with short runs such as the operation of cranes or urban transport. From these curves, with little calculation, the values of such quantities as mean speed, power consumption and temperature rise of motor when operating on a statistically specified schedule, may be estimated for various combinations of motor windings, gear ratios, accelerating currents and the other quantities open to choice.

The procedure is based on the recognition of the conditions under which the characteristic curves of the motor and the speed/time curves in operation are of similar shape. One condition for this is that the magnetization curves of the motors should be of the same shape. For classes of motor and operation satisfying these conditions for similarity, various 'required' quantities may be given by single curves with scale factors. It is shown that the curves as drawn (i.e. with scale factors of unity) are the characteristics of a hypothetical sample or 'model' of the class of drives. Such models are not necessarily realizable physically.

The principles are illustrated by the development of curves giving run times, current consumption, motor-heating coefficients and other quantities that result from the operation of series-wound motors having magnetization curves of the shape found to be characteristic for modern mill-type motors. The application of these curves is briefly illustrated by consideration of the choice of a motor for a long-travel motion on a crane.

The data presented are intended to be illustrative rather than complete and to suggest means by which users and designers may prepare data adapted to their particular needs and to the particular classes of motor in question.


## LIST OF PRINCIPAL SYMBOLS

*Note.*—Symbols of quantities with a suffix 1, as $\phi_1$, $u_1$, $t_1$, etc., are scale factors. A table of these scale factors, with their values in terms of the primary variables is given in the Appendix.

$A$ = Magnetomotive force of series winding, AT.
$b$ = Linear retardation during electrical braking, ft/sec$^2$.
$H$ = Heating value ($=\int\theta_p dt$), deg-C-sec.
$i_c$ = Current of motor on a continuous temperature test, amp.
$i_1$ = Current at representative point on magnetization curve ($=A_1/N$), amp.
$K_a$ = Ratio of accelerating current to current $i_1$ at representative point.
$K_e$ = Factor giving rotational e.m.f., volts per radian per second and per unit flux.
$K_f$ = Ratio of frictional retardation to acceleration during starting period $[=P_f/(P_a - P_f)]$.
$K_r$ = Ratio of resistive drop in motor circuit at representative current to the supply voltage ($= ri_1/V$).
$K_s$ = Cooling correction factor.
$l$ = Length of run, ft.
$M$ = Motor torque, lb-ft.
$m$ = Mass of load (including allowance for rotating parts), lb.
$N$ = Number of turns of series winding.
$P$ = Tractive effort at load, lb.
$P_a$ = Accelerating force, lb.
$P_b$ = Retarding force during plugging, lb.
$P_f$ = Frictional or tractive resistance, lb.
$Q$ = 'Current consumption', amp-sec.
$R$ = Ratio of linear to angular velocity ($=u/\omega$), ft.
$r$ = Motor circuit resistance, ohms.
$u$ = Speed of load, ft/sec.
$V$ = Line voltage, volts.
$W$ = Weight equivalent of mass of load, lb wt.
$\theta_c$ = Temperature rise on continuous test, deg C.
$\theta_p$ = 'Potential' temperature rise, deg C.
$\omega$ = Angular speed of motor shaft, rad/sec.

Prof. Tustin, formerly at Birmingham University, is Professor of Electrical Engineering at the Imperial College of Science and Technology, University of London.
Mr. Nettell, formerly with the British Iron and Steel Research Association, is with Powers Samas Accounting Machines, Ltd.
Mr. Solt, formerly with the British Iron and Steel Research Association, is with the Brush Electrical Engineering Co., Ltd.

## (1) REQUIREMENTS IN SELECTION OF MOTORS FOR TRACTION-TYPE DUTIES

The selection of an economical motor for intermittent short-run duties, such as on cranes, roller-table drives in steelworks, vehicles on frequent-stop services and the like, is a laborious and confusing task, because of the large number of variables and the way in which requirements conflict and interlock. The motor must have a speed/torque characteristic such that it will perform the duty, and in performing the duty it must not get too hot. The gear ratio is often open to choice. In some cases the possibilities of selection of the number of turns of the series winding or of the armature winding or of both must be considered. The time and labour required to assess the performance of a variety of possible choices of motor or variants of design is one reason why, at the present time, many motors are applied uneconomically.

A basic source of difficulty is that relationships are involved that are non-linear, some of which, such as the speed/torque relationship, cannot be expressed in any convenient mathematical form, so that an analytical solution is not practicable. The most that can usually be done is to seek to extend the scope of calculated or empirical data to the widest possible field of applications, using the principles of similarity and dimensional analysis, and it is the purpose of the paper to show how this may be done in the motor-application problem. Several authors have made use of principles of similarity to deal with motor applications,[1, 2] but the approach in this paper is more general than in previous discussions.

## (2) THE NOTION OF AN AFFINE SET OF CURVES, AND OF A 'REPRESENTATIVE POINT'

A family of curves are said to be 'affine' if the individual curves differ only by change of scale of one or both sets of co-ordinates. The term 'similar' sometimes used in this sense is properly applicable only when both scales change in the same proportion. Any one curve of an affine set, together with the two scale ratios for ordinates and abscissae respectively, specifies some other

curve from the set. It is convenient to think of each curve as having one point distinguished as a 'representative point' placed in each case in the same relationship to the curve as a whole. Then a particular curve of the set is distinguished by the co-ordinates of its representative point.

From a complete affine set it is always possible to select the particular curve for which the co-ordinates of the representative points are $x = 1$, $y = 1$. If the relationship satisfied by this curve is designated $y = F(x)$, then any other curve of the set, say the one for which the co-ordinates of the representative point are $x = x_1$, $y = y_1$, has ordinates and abscissae of corresponding points multiplied by $x_1$ and $y_1$ respectively. It therefore satisfies the relationship $y/y_1 = F(x/x_1)$. Conversely, if a relationship between two quantities $x$ and $y$ can be expressed in the form $y/y_1 = F(x/x_1)$, with $x_1$ and $y_1$ independent of $x$ and $y$, then the relationship is represented by a set of affine curves, and $x_1$ and $y_1$ are scale factors relating the curve $y/y_1 = F(x/x_1)$ to the 'unit' curve $y = F(x)$. If $F(1) = 1$ and this point on $F(x)$ is regarded as a representative point, then $x_1$ and $y_1$ are the co-ordinates of the representative point on other curves of the set.

Consider, for example, the forms of on-load magnetization curves of electrical machines, such as that shown in Fig. 1.

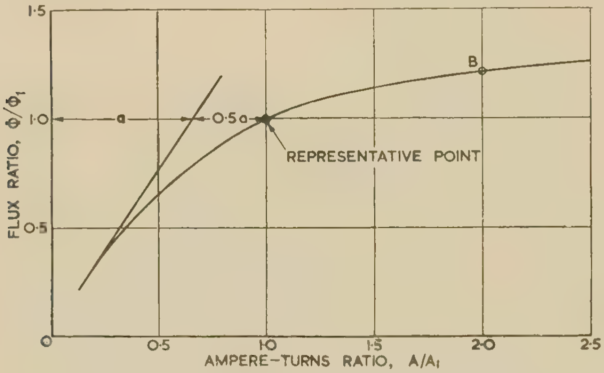


**Fig. 1.**—Flux ratio as a function of ampere-turns ratio.

Such curves have an approximately straight part, the air-gap line, for low values of ampere-turns $A$, and another approximately straight part for very large values of $A$, because ultimately the incremental permeance of iron approaches that of air. These straight parts are joined by the part referred to as the knee of the curve. Magnetization curves may differ in the relative slopes of the two linear parts and in the form and extent of the knee, but for machines of the same general class these features differ only moderately. For curves in which these features are alike the only remaining difference is in the scales of m.m.f. and of flux, and all such curves belong to an affine set.

There is no point on a magnetization curve distinguishable in a way that would make it naturally suitable to serve as a representative point, for the position of the knee is not precisely defined. A representative point may, however, be defined in various ways, for example as the point at which the ampere-turns are greater in some specified ratio (say 1·5) than would be given by prolongation of the air-gap line. If one considers a number of different motors, possibly of a wide range of values of flux and m.m.f., having possibly also slight differences in the shapes of their magnetization curves, one may draw these curves to such scales that the representative points from all of them coincide. The curves so drawn may not completely coincide at all points, but they coincide at their representative points. If an 'average' curve is then drawn the agreement with the original curves will be close near the representative point. For this reason the representative point may with advantage be placed in the region of the curve that is most significant for the purpose in view. In the applications considered here, the most significant region of the magnetization curve is that involved during acceleration, and for this reason the method of defining a representative point suggested above using the ratio 1·5 is suitable.

The effective identity of shape of the magnetization curves for lines of motors of a similar kind is already recognized in some design departments by the use of a common curve on a percentage basis. It has also been found that such curves as used by different manufacturers are in close agreement. Fig. 1 gives such a magnetization curve for modern mill-type motors.

## (3) CONDITIONS UNDER WHICH THE SPEED/CURRENT CHARACTERISTICS OF SERIES-WOUND MOTORS ARE AFFINE

Given the magnetization curve for a motor, the speed/current characteristic when series connected may be derived by the relationship

$$2\pi n = \omega = \frac{V - ri}{K_e\Phi} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $V$ is the supply voltage, $r$ the motor circuit resistance and $K_e$ a factor for rotational e.m.f. defined as volts per radian per second per unit flux. If the magnetization curve is one from an affine set expressed by

$$\frac{\Phi}{\Phi_1} = F_1\frac{A}{A_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where $\Phi_1$ and $A_1$ are the co-ordinates of a representative point defined, say, as suggested, then the condition that the speed/current curves are members of an affine set is that the resistive drop is in each case the same fraction of $V$ when $A = A_1$. This is shown formally and the scale factors are established by inserting in eqn. (1) the value of $\Phi$ from eqn. (2), which gives, for $N$ series turns and $i = A/N$,

$$\omega = \frac{V - rA/N}{K_e\Phi_1F_1(A/A_1)} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Writing $K_r$ for the ratio of the resistive drop at the representative current to the supply voltage,

i.e. $$K_r = \frac{rA_1/N}{V} = \frac{rA}{N}\frac{A_1}{VA}$$

then, in eqn. (3), $rA/N$ may be replaced by $K_rVA/A_1$ and

$$\omega = \frac{V(1 - K_rA/A_1)}{K_e\phi_1F_1(A/A_1)} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

or $$\frac{\omega}{V/K_e\Phi_1} = \frac{1 - K_rA/A_1}{F_1(A/A_1)} \quad . \quad . \quad . \quad . \quad (5)$$

The right-hand side is a function only of $A/A_1$ for any particular value of $K_r$. Writing this function as $F_2(A/A_1)$ and writing $V/K_e\Phi_1 = \omega_1$ and $A_1/N = i$, where $N$ is the number of series turns, the speed/current relationship is

$$\omega/\omega_1 = F_2(A/A_1) = F_2(i/i_1) \quad . \quad . \quad . \quad .$$

$$= \frac{1 - K_r(i/i_1)}{F_1(i/i_1)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The current, i.e. $i_1 = A_1/N$ will be referred to as the representative current.

By eqn. (6), the curves relating $\omega$ and $i$ are an affine set with the scale factors $i_1 = A_1/N$ for current and $\omega_1 = V/K_e\Phi_1$ for speed. The ratio $V/K_e\Phi_1$ has the dimension of angular speed and is the angular speed at which the back e.m.f. is $V$ when t

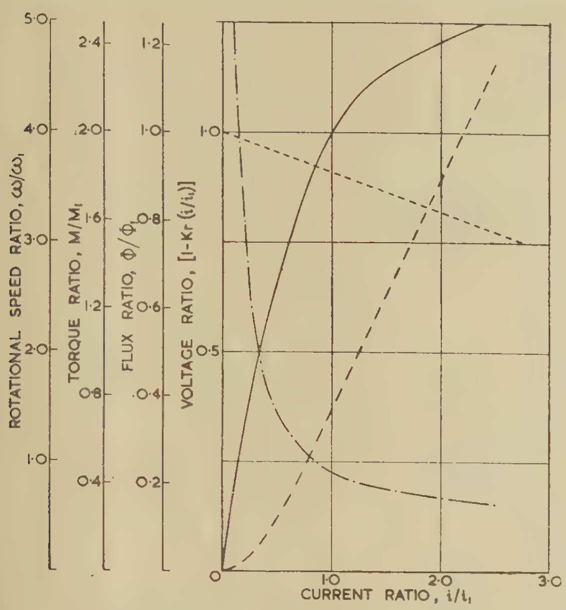


**Fig. 2.**—Flux, speed, torque and back-e.m.f. ratios as functions of current ratio.

- - - - - - - Back-e.m.f. ratio.
— . — . — Speed ratio.
———— Flux ratio.
— — — — Torque ratio.

$\omega_1 = V/K_e\Phi_1$ radians per second.
$M_1 = Vi_1/\omega_1 = K_e\Phi_1 i_1$ pound-feet.
$K_r = ri/V$.

flux is $\Phi_1$. The speed at which a motor runs when the excitation corresponds to the representative point is that at which the back-e.m.f. is $V(1 - K_r)$. Thus, for this condition

$$\omega/\omega_1 = 1 - K_r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Fig. 2 shows the function $F_2(i/i_1) = \omega/\omega_1$ calculated for the particular form of magnetization curve given by Fig. 1 and for a value of $K_r$ of 0·088, which was representative for the middle range of the examples of motors considered.

*The Concept of a 'Model' whose Characteristics are the 'Unit' Curves.*—One can conceive of a motor for which the magnetization curve is $\Phi = \mathrm{F}_1(A)$, and the speed/current curve is $\omega = \mathrm{F}_2(i)$, i.e. the scale factors $\Phi_1$, $A_1$, $\omega_1$ and $i_1$ are all unity. Such a motor with $\Phi_1 = 1$ maxwell would not be physically realizable, but there is usefulness in the idea of such a 'model' irrespective of physical realizability. The complete set of characteristic curves, including the tractive-effort/speed and the speed/time curves which will be discussed below, may be thought of as the characteristic curves of this 'model'. They may be calculated, starting from the 'unit' magnetization curve, by using any of the arithmetical or graphical procedures that would be used in the case of a real motor and load. The numerical values in which the co-ordinates are scaled appear very unfamiliar, because the calculation refers to an imaginary motor outside the real range, but this is put right by the scale factors. The 'model' is conceptual or mathematical and is defined by being supposed to satisfy the relevant equations, with the scale factors unity; i.e. to satisfy eqn. (5), with $A_1$ and $\Phi_1$ unity. The curves calculated for the model are generalized for application to any motor or drive of the class in question by remarking the scales in ratios instead of actual values; e.g. with $\Phi/\Phi_1$ replacing $\Phi$, $A/A_1$ replacing $A$, $\omega/\omega_1$ replacing $\omega$, and so on. This produces the same result as calculating the curves directly from the expressions in ratios [such as eqn. (6)].

The conditions so far required to specify the 'model' are that $\Phi_1$, $A_1$, $\omega_1$ $(=V/K_e\Phi_1)$ and $i_1(=A_1/N)$ are all unity. It follows that $V/K_e = 1$, $N = 1$. Other requirements will be added later.

### (4) THE CORRESPONDING AFFINE SET OF TORQUE/CURRENT CHARACTERISTICS

The torque in dyne-cm is given by

$$M^1 = 10^7 K_e \Phi i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

or in lb-ft by

$$M = 0{\cdot}737 K_e \Phi i \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

If the magnetization curves and the speed/current curves are affine sets, the torque/current curves are also an affine set. This is shown, and the scale factors are determined, by inserting in eqn. (9) the value of $\Phi$ from eqn. (2), giving

$$M = 0{\cdot}737 K_e \Phi_1 \mathrm{F}_1(i/i_1) i \quad . \quad . \quad . \quad . \quad (10)$$

or

$$M/K_e\Phi_1 i_1 = 0{\cdot}737 i/i_1 \mathrm{F}_1(i/i_1) \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where the right-hand side is a function of $i/i_1$, say $\mathrm{F}_3(i/i_1)$, and writing $K_e\Phi_1 i_1 = M_1$

$$M/M_1 = \mathrm{F}_3(i/i_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

so that the curves are affine with scale factors $M_1 = K_e\Phi_1 A_1/N = Vi_1/\omega_1$ and $i_1 = A_1/N$. In Fig. 2 the function $\mathrm{F}_3(i/i_1)$ also is shown.

The curve for which the scale factors are unity is $M = F_3(i)$, and by eqn. (11) this is the torque/current curve for the same 'model' if, in addition to the previous requirements, $A_1 = 1$, $\Phi_1 = 1$, $N = 1$ and $V/K_e = 1$, the further specification is added that $K_e = 1$, which implies $V = 1$ volt.

The point on the curve for the model that corresponds with the original representative point is $M = 0{\cdot}737$ lb-ft, $i = 1$ amp.

### (5) THE SPEED/TORQUE CHARACTERISTICS

The relationship of torque to speed follows simply from the expressions for torque and speed respectively as functions of current, namely eqns. (6) and (12).

Representing the former in the inverse form

$$i/i_1 = \mathrm{F}_2^{-1}(\omega/\omega_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

and inserting this in eqn. (12) gives

$$M/M_1 = F_3[F_2^{-1}(\omega/\omega_1)] \text{ a function of } \omega/\omega_1$$
$$= F_4(\omega/\omega_1), \text{ say} \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

The function $F_4(\omega/\omega_1)$ corresponding to Figs. 1 and 2 is shown as Fig. 3.

The speed/torque curves are thus an affine set with the same scale factors $M_1$ and $\omega_1$ as the previous curves. The torque/speed curve for the model is $M = \mathrm{F}_4(\omega)$, and the torque and speed for the model corresponding to the representative point are $M = 0{\cdot}737$ lb-ft, and $\omega = 1 - K_r$ rad/sec.

### (6) THE TRACTIVE-EFFORT/SPEED OR LOAD CHARACTERISTICS

For motors used to give linear displacement to a load it is usual to draw curves that relate the linear speed of the load, say in feet per second, to the 'tractive effort' due to the motor, say in pounds. These curves differ from the $M/\omega$ curves only in scales and are therefore also an affine set. It is convenient to express the overall velocity ratio as

$$\frac{\text{Speed of load, ft/sec}}{\text{Rotational speed of motor, rad/sec}} = \frac{u}{\omega} = R \quad . \quad (15)$$

so that $R$ is the radius in feet at which the peripheral speed of the

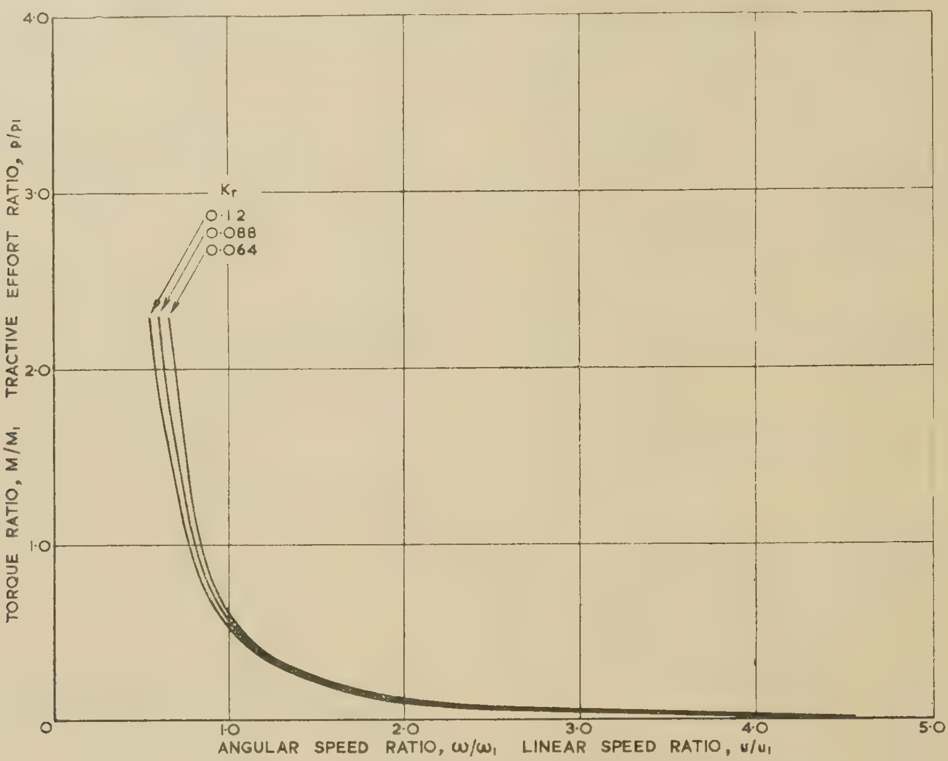


Fig. 3.—Speed ratio (linear or angular) as a function of torque or force ratio, for several values of $K_r$.

$\omega_1 = V/K_e\Phi_1$ radians per second.
$M_1 = Vi_1/\omega_1$ pound-feet.
$P_1 = M_1/R$ pounds.
$u_1 = \omega_1 R$ feet per second.

motor shaft is equal to the load speed. Then if $u$ is the linear speed and $P$ the tractive effort, $\omega = u/R$ and $M = PR$, and substituting these values in eqn. (14) gives

$$\frac{P}{M_1/R} = \mathrm{F}_4\frac{u}{(\omega_1 R)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

which, writing $M_1/R = P_1$, $\omega_1 R = u_1$, gives

$$P/P_1 = \mathrm{F}_4(u/u_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The scale factors are $P_1 = K_e\Phi_1 A_1/NR = Vi_1/\omega_1 R$ for tractive effort and $u_1 = RV/K_e\Phi_1 = R\omega_1$ for speed. The curve for the model, imposing the further specification that $R = 1$ ft, is $P = \mathrm{F}_4(u)$ and is the same as the curve for angular-speed/torque (Fig. 3). The point corresponding to the representative point is $P = 0{\cdot}737$ lb, $u = 1 - K_r$ ft/sec. Additional curves are given in Fig. 3 for $K_r = 0{\cdot}12$ and $0{\cdot}064$ as well as $0{\cdot}088$.

Thus all the usual characteristic curves of all series-wound motors are affine sets and may be represented by sample curves and scale factors, provided only that the magnetization curves are affine and that the relative resistive drop $K_r$ is the same at the same representative point. Further, as sample curves the set of curves may be taken, calculated for a hypothetical model for which the value unity is given to $\Phi_1$, $A_1$, $N$, $K_e$, $V$ and $R$. This set of sample curves may be turned into universal curves (for a given class of magnetization curves and a given value of $K_r$) by relabelling the co-ordinates with the ratios to the scale factors that have been stated. Fig. 4 shows the relationship between the various characteristic curves for the model. The points corresponding to the representative point are marked on each. A list of scale factors, including those for other characteristics to be considered later, is given for reference in the Appendix. Usually the most convenient parameters in which the scale factors can be expressed are $i_1$ and $\omega_1$, and the values in this form are given in the final column of the list.

## (7) THE SPEED/TIME AND CURRENT/TIME CURVES FOR A RUN OR A START-TO-STOP OPERATION

The conditions will now be considered under which the speed/time and current/time curves form affine sets when the motor is operated to move a load having inertia, against frictional or other resistance.

These characteristics are of a different type from those so far considered, in that they are not inherent in a given motor but depend upon the motor being operated in some definite manner which must be specified. The only mode of operation that will be considered here is the common one in which a starting resistance is cut out in such a way that the current is maintained constant and the load is accelerated uniformly until full line voltage is reached at the motor terminals. The tractive resistance will be assumed constant and the braking retardation uniform. The general appearance of such speed/time and current/time curves is the familiar one of the form indicated at the right-hand side of Fig. 4.

For a set of speed/time curves of this kind to be affine the following geometrical conditions suffice:

(a) The constant acceleration during starting, the retardation during coasting, and the retardation during braking must be in the same ratio. These are the slopes of the straight-line parts of the diagram.

(b) The values of acceleration after full line voltage is reached must, at corresponding points, have the same ratios to the starting acceleration.

(c) Braking must commence at a time that is the same multiple of the time required to reach full line voltage.

(d) If there is coasting, it must also commence at the same multiple of the time required to reach full line voltage.

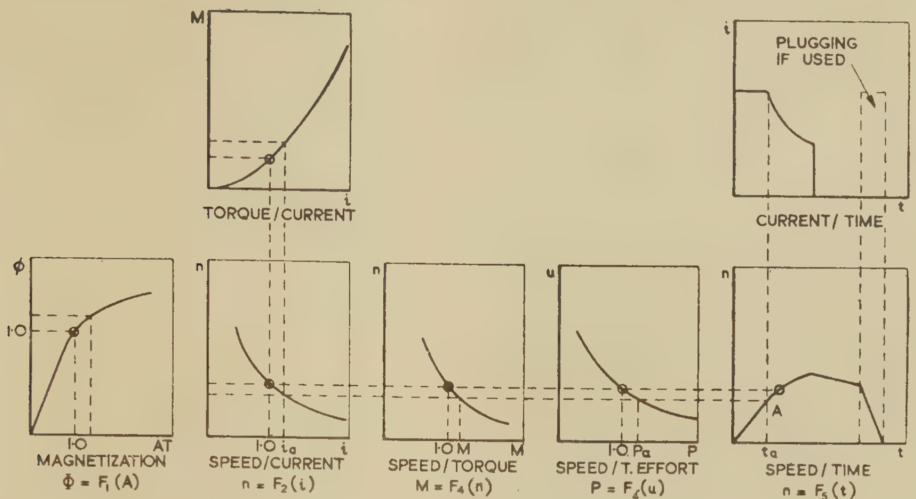


Fig. 4.—Relationship between various characteristics for the model motor.
⊙ Representative point.

Condition (*a*) requires that the frictional or tractive resistance should be in the same ratio to the tractive effort at the accelerating current. This is an inconvenient requirement, because it prevents the direct calculation of required quantities for fixed values of tractive resistance and various values of accelerating current. A process of replotting is unavoidable when such information is required.

To secure condition (*b*) requires that the speed/tractive-effort curves should be affine over the range of speeds involved, and that the points at which full line voltage is reached should be corresponding points. This in turn implies that the accelerating current must be in the same ratio in all cases to the representative current $i_1 = A_1/N$.

## (8) THE AFFINE SETS OF SPEED/TIME AND SPEED/CURRENT CURVES CORRESPONDING TO A GIVEN SET OF AFFINE SPEED/TRACTIVE-EFFORT CURVES

Subject to the restrictions just mentioned, the speed/time and current/time curves may be specified by sample curves and scale factors. No simple mathematical expression exists for a speed/time curve as a whole, but it is sufficient to consider the initial straight-line section of the speed-time graph, terminating at the point where full line voltage is reached, since the conditions specified above are those for affinity and ensure that the rest of the diagram changes in scales with this section. The point at which full voltage is reached, shown at A in Fig. 4, may be regarded as a representative point for a speed/time curve; its scale factors may be obtained as follows by consideration of the co-ordinates of this point, which will for convenience be called the 'first full-voltage point'.

The value of the starting acceleration involves the frictional resistance. The ratio between the slope of the coasting line (due to frictional resistance alone) and that of the straight-line part during starting must be restricted to the same value as a condition that the curves be affine. Let this ratio be $K_f$. Then $K_f$ is the ratio between $P_f$, the frictional force, and $P = P_a - P_f$, where $P_a$ is the tractive effort, and $P$ the net accelerating force;

i.e. $$K_f = P_f/P \text{ and } P = P_a - P_f = P_a - K_f P \quad . \quad . \quad (18)$$

from which $P = P_a/(1 + K_f)$; i.e. the initial acceleration is reduced on account of the frictional resistance by the factor $1/(1 + K_f)$.

Let the effective mass at the load be $m = W/g$. Any component due to the kinetic energy of rotating parts will be supposed already included in $m$ and $W$. The linear acceleration is

$$a = \frac{P}{m} = \frac{P_a g}{(1 + K_f)W} \quad . \quad . \quad . \quad . \quad . \quad (19)$$

$g$ being the acceleration due to gravity (32·2 ft/sec$^2$). If the speed corresponding to $P_a$ on the speed/tractive effort curve for full line voltage is $u_a$, the time in seconds required to reach line voltage (speed $u_a$) is

$$t_a = \frac{u_a}{a} = \frac{u_a(1 + K_f)W}{P_a g} \quad . \quad . \quad . \quad . \quad (20)$$

The speed $u_a$ and the time $t_a$ are the co-ordinates of the representative point (first full-voltage point). Each position of this point corresponds to one of the family of affine speed/time curves.

## (9) THE SCALE FACTORS FOR SPEED/TIME AND SPEED/CURRENT CURVES

As a sample curve, one may conveniently take that for the 'model', with the additional specification that the mass of the load corresponds to a weight of 1 lb. If the accelerating current and the speed and torque corresponding to it on the curves for the model are $i_{a1}, u_{a1}, P_{a1}$, the first full-voltage point for the model is at $u_{a1}$ and $t_{a1} = u_{a1}(1 + K_f)/P_{a1}g$, since $W = 1$.

The co-ordinate ratios, or scale factors, by which the curve for the actual system is related to that of the model are therefore

*For time.* $$t_1 = \frac{t_a}{t_{a1}} = \frac{u_a(1 + K_f)W}{P_a g} \bigg/ \frac{u_{a1}(1 + K_f)}{P_{a1} g} = \frac{u_a P_{a1} W}{u_{a1} P_a} = \frac{u_1 W}{P_1}$$

*For speed.* $u_1 = \dfrac{u_a}{u_{a1}}$, as for the speed/tractive-effort curve.

Similarly, the current is scaled up by $i_1$. Distances, since they are represented by areas under the speed/time curves, have values greater than for the model, on account of the extension both in the time axis and the speed axis, so that the scale factor for distance is that for speed multiplied by that for time;

i.e. $$l_1 = \frac{u_1^2 W}{P_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

The scale factor for ampere-seconds ('current consumption') is the product of that for current and that for time;

i.e. $$Q_1 = \frac{i_1 u_1 W}{P_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (22)$$

The energy consumption is $QV$, and since $V$ for the model is unity the scale factor is

$$\frac{Vi_1 u_1 W}{P_1}$$

These scale factors, and their equivalents in terms of the basic parameters, have been listed for reference in the Appendix.

## (10) THE HEATING EFFECT OF A RUN, AND THE SCALE FACTORS FOR GENERALIZING THE HEATING VALUE FOR A SET OF AFFINE CURRENT/TIME CURVES

It is required in selecting a motor to estimate the temperature its windings will reach when it is used to perform the required service. It is shown in a companion paper[4] that the temperature rise of the windings will ultimately reach a value that is the mean of the 'potential temperature rises' corresponding to the various operating conditions through which the motor passes, corrected by a suitable factor if the schedule speed is other than the 'standard' speed for which the potential temperature rises have been recorded. Fig. 5 shows the values of potential temperature

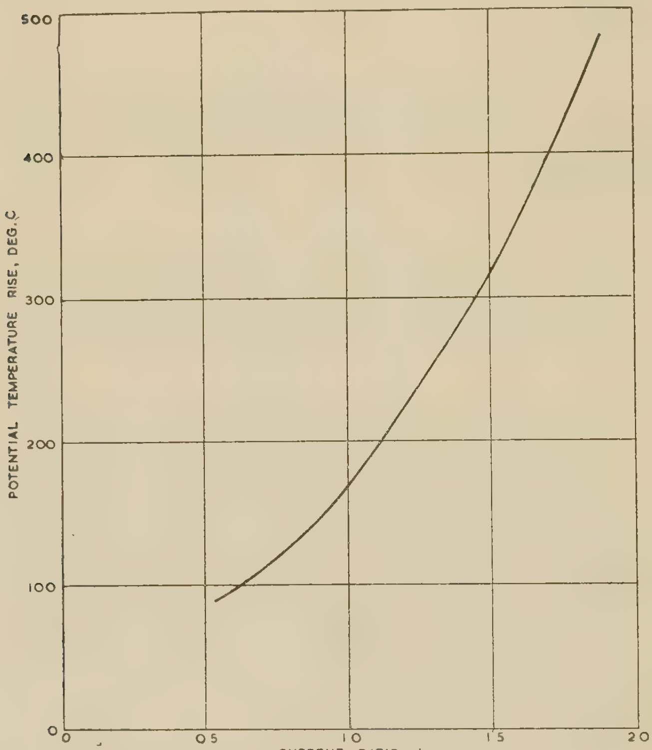


Fig. 5.—Potential temperature rise measured on a particular (25 h.p. totally-enclosed) motor $x$ as a function of $i/i_1$.
For this motor, $i_1$ was 74·4 amp and the standard speed was 590 r.p.m.

rise as a function of current, and Fig. 6 the factor for correction for speed as a function of speed ratio, obtained by measurement on a 25 h.p. series-wound mill-type motor.

The temperature rise that results from a given schedule of runs is thus given by

$$\theta = K_s \frac{\Sigma \int \theta_p dt}{\Sigma t} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (23)$$

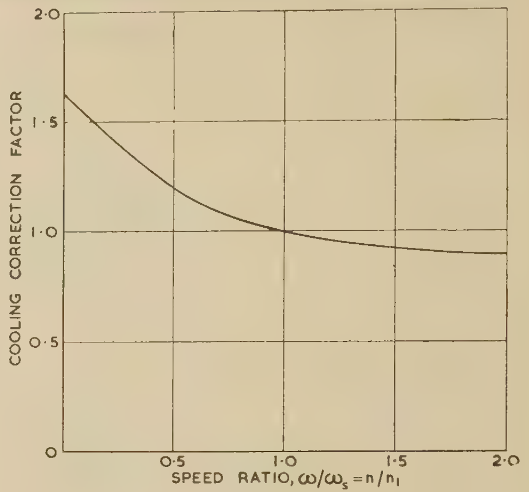


Fig. 6.—Cooling correction factor versus speed ratio.
Standard speed = 590 r.p.m.

where $\theta_p$ is the potential temperature rise (a function of curren where the integral sign represents integration over one run, $\Sigma$ represents summation for all the runs, $\Sigma t$ is the total time, an $K_s$ the factor for correction according to the ratio schedule speed/standard-speed.

If the required two heating characteristics—the potential tem perature rise characteristic and the characteristic of correctio for speed—have been obtained by measurement on one repre sentative motor for a line or type, then these data, together wit a continuous rating for any motor under consideration, suffic to generalize the calculation made for any one current/time curv to any other affine to it, and so to estimate the temperature ris of any motor of that class on any schedule, on the followin further assumptions:

(i) That the potential temperature rise characteristics all have th same shape, and are similarly disposed in relation to the representativ point on the saturation curve, for all the motors under consideratio

(ii) That the factor for correction for speed is the same for all th motors for the same speed ratio.

The justification for assumption (i) may be seen on notin that the potential temperature rise characteristic represents th dependence of 'effective loss' on current. If the only loss wer $ri^2$ the curve would be a parabola, and all such curves would b affine. The departure from a parabolic form is due to othe losses, the most important of which, such as iron and stray losse are associated with the degree of saturation of the iron and s are located by the representative point. Some of the loss independent of current, and some (such as that due to brus contact drop) is proportional to current instead of to its squar So long as these components are in the same proportion corresponding points they result in the same shape of curv The assumption of a similar form of distribution of effecti losses relative to the representative point on the saturation cur may be considered as a substantial and necessary improveme on the familiar 'r.m.s. current' method of estimation, whic assumes the curve to be a parabola and omits the effect of spee altogether. No assumption is made here about the actu magnitude of the effective loss or the relative heat dissipation c the various motors, since these factors will be introduced t relating the value estimated to the continuous rating of the mot under consideration.

This is illustrated in Fig. 7. Suppose a motor is under co sideration, designated by suffix $y$, and that the result of a co tinuous test is available, in which a current $i_{yc}$ at speed $n_{yc}$ ga a temperature rise $\theta_{yc}$.

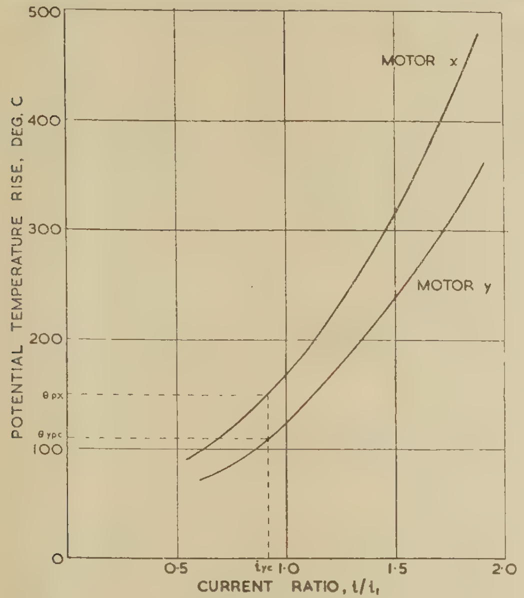


Fig. 7.—Similarity of potential temperature curves.
Standard speed = 590 r.p.m.

Let there be available also, from tests on some other motor, designated by suffix $x$, a potential-temperature characteristic, with $\theta_{px}$ plotted as a function of $i_x/i_{1x}$ ($i_{1x}$ being the representative current). The hypothesis is that the potential-temperature curve for motor $y$, when plotted as a function of $i_y/i_{1y}$ is a similar curve, i.e. of the same shape but possibly higher or lower by some factor. To fix the ordinate scale, one has the result of the continuous test. By definition, the temperature rise on a continuous test, corrected to the standard speed, is the potential temperature rise. From the speed-correction characteristic may be read the factor $K_{sc}$, by which the temperature rise is multiplied because the speed is $n_{sc}$ instead of the standard speed, so that the potential temperature for motor $y$ at $i/i_1 = i_{yc}/i_{1y}$ is $\theta_{ycp} = \theta_{yc}/K_{sc}$. The ordinate of the potential temperature rise characteristic for motor $x$ at the same value of $i/i_1$ may be read; say it is $\theta_{xcp}$. Then the scale ratio between ordinates of the potential temperature characteristics is $K_{xy} = \theta_{ycp}/\theta_{xcp}$. By including this factor, one takes account of the different ratios of dissipation to loss on different motors.

The quantity required is $\int\theta_p$ over a run, which will be designated $H$, the 'heating effect'. A convenient procedure is to calculate this value for the 'model' drive (reading $\theta_p$ from the curve for motor $x$ for each value of $i/i_1$, and integrating graphically). So far as the ordinate scale is concerned, the result must be multiplied by $K_{xy}$, but there is no other ordinate scale factor because the similar current/time curves are all similarly related to $i_1$.

The current/time curves for various drives are extended in time as compared with that for the model by the scale factors for time $t_1$, so that finally the scale factor for $H = \int\theta_p dt$ (area under the curve) is $t_1 K_{xy}$.

It should be noted that the values of $H$ obtained are not absolute but depend on the particular motor ($x$) for which potential temperatures were available. This is put right by the inclusion of the factor $K_{xy}$ as a scale factor.

Reference to the example given in Section 14 may make the above procedure clearer.

## (11) CURVES GIVING REQUIRED VALUES FOR SYSTEMATICALLY VARIED SHAPES OF RUN CURVES

By the use of the scale factors, calculations made from the speed/time and current/time curves for the 'model' are made applicable to all those drives for which the curves are affine to those of the model. The speed/time and current/time curves are affine only for the conditions listed in Section 7. When these conditions are varied, e.g. when, say, some other ratio of accelerating current to the current at the representative point is used, or some other ratio of friction to accelerating force occurs, the shapes of these curves are different, and distinct calculations must be made for each different combination. The results of such calculations, made for the model, may be recorded as curves, relating a required quantity (e.g. current consumption) to the quantity which varies the shape of the speed/time curve (e.g. length of run). By relabelling the scales by ratios of the quantities to the appropriate scale factors, each point on such curves represents the relationship for all of a set of affine curves. The results of a systematic variation of shape by varying one ratio can be recorded by a single curve, and the results of systematic variation of two ratios by a family of curves, or one curve sheet.

The variety of distinctive shapes of run curve that are possible, even confining attention to series-wound motors, is so large, and the quantities or relationships of practical interest are so numerous, that complete mapping of the possible behaviour is scarcely practicable, and a careful choice must be made of the particular variations to be explored.

In this paper no more can be done than to give a few examples illustrating the possibilities. A selection of such performance curves for series motors is given in Figs. 8–13. They are based on families of speed/time and speed/current curves, of which Fig. 8 shows representative samples. The effect of variation of the parameters has been shown at each stage in developing the run curves, but, for simplicity, only one value of the parameter has been carried forward to subsequent stages. For example, Fig. 3 shows the effect of the variation of the ratio $K_r$ of resistive e.m.f. to line e.m.f., but only the value $K_r = 0{\cdot}088$ has been used in the construction of the ensuing curves. The following representative values of parameters have been selected in this way and are assumed in the sample curves for current consumption and motor heating to be given later in Figs. 10 and 11:

$K_r = 0{\cdot}088$
$K_f = 0{\cdot}3$
Accelerating time = 0·6 of total run time.
Starting current = Twice representative current.
Plugging current = Accelerating current.

In Fig. 8, three only of a sequence of speed/time curves are drawn, associated with the acceleration curve for $K_f = 0{\cdot}3$, in which the current-on time is kept a constant fraction (0·6) of the run time and the run time is varied. This involves a progressive increase in the proportion of time spent in coasting. Figs. 9–12 give the dependence of run time, mean speed, current consumption and heating value on the distance, for runs in which the current-on time is a constant fraction ($t_m/t_r$) of the run time. This ratio is 0·6, except that in Fig. 13 the results of two smaller ratios, 0·4 and 0·5, are given for comparison, and a separate curve, marked 'no coasting', is given for the cases in which the current is maintained until braking commences. The ratio of current-on time to total time is here different for each point in the curve.

It would, of course, have been equally possible to display the results for a set of runs of various lengths keeping the proportion of coasting time constant, in which case the current-on time would necessarily change systematically in proportion to the length of run, and the curves would have been slightly different.

It will be understood that all the ratios or parameters that affect the shape of the run curve are always kept constant, except for those that are the co-ordinate values in the particular case, or that

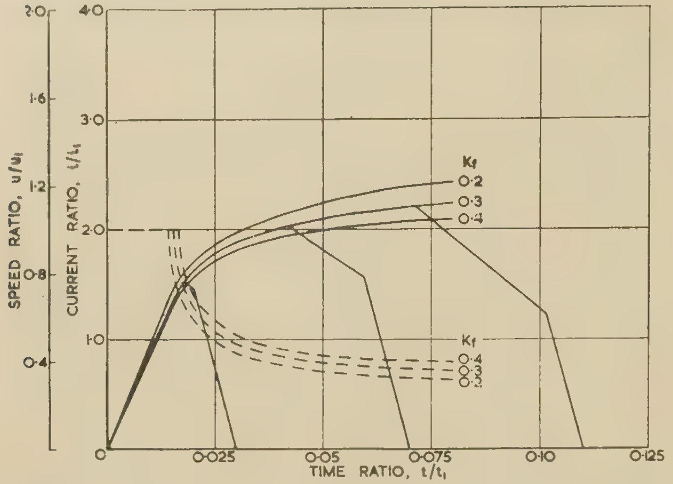


**Fig. 8.**—Speed ratio and current ratio as functions of time ratio for various values of $K_f$.

$i_a/i_1 = 2$.
$t_m/t_r = 0\cdot6$.
$K_r = 0\cdot088$.
$u_1 = R\omega_1$ feet per second.
$t_1 = u_1W/P_1 = R^2\omega_1^2W/Vi_1 = u_1^2W/Vi_1$ seconds.

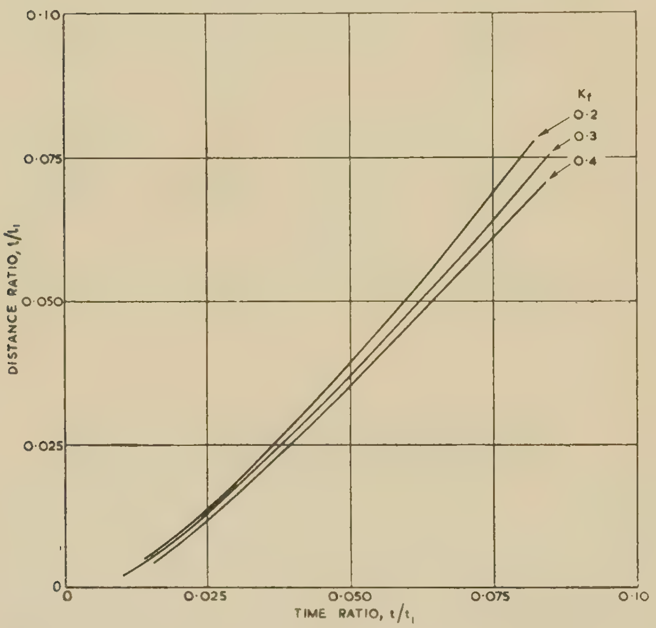


**Fig. 9.**—Distance ratio as a function of time ratio if current were maintained.

$i_a = 2i_1$.
$K_r = 0\cdot088$.
$l_1 = R^3\omega_1^3W/Vi_1$ feet.
$t_1 = u_1^2W/Vi_1$ seconds.

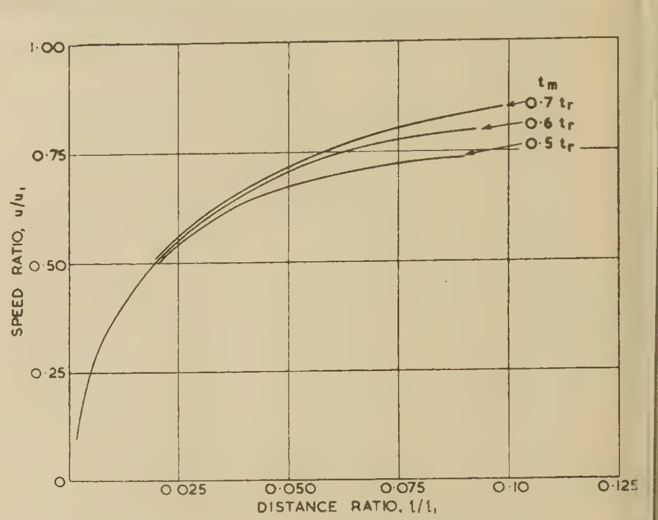


**Fig. 10.**—Mean speed ratio as a function of distance ratio for sever values of current-on-time/motoring-run-time, $t_m/t_r$.

$l_1 = u_1^3W/Vi_1$ feet.
$u_1 = R\omega_1$ feet per second.

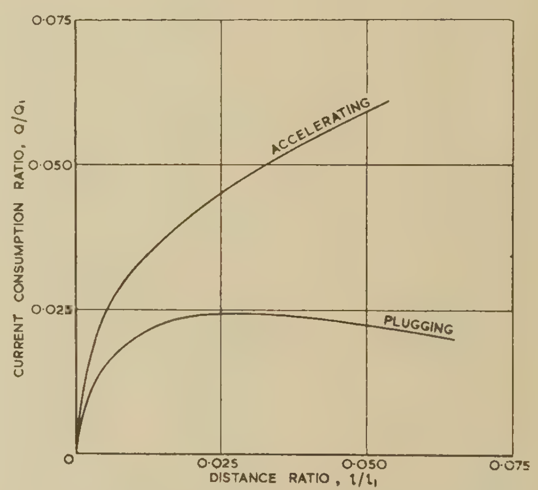


**Fig. 11.**—Current consumption ratio (motoring and braking separate as a function of start-to-stop distance ratio.

$K_f = 0\cdot3$.
$l_1 = u_1^3W/Vi_1$ feet.
$Q_1 = u_1^2W/V$ ampere-seconds.
$u_1 = \omega_1R$ feet per second.

have different values for different distinct curves, as marked on the diagrams.

In particular, the ratio of accelerating current to representative current is assumed to have the value 2 for all the curves presented, corresponding to the point on the magnetization curve marked as B in Fig. 1.

The following points should also be noted:

(*a*) Single-motor control has been assumed. Where series-parallel control is used the current consumption is reduced.

(*b*) No allowance has been made for torque reduction due to iron losses and frictional losses in the motor. These should be taken in account as an addition to frictional resistance.

(*c*) A much more extensive set of such data curves is needed i practical use and as a basis for the various comparisons that a required to select the most economical drive and method of operatio Such data are best provided by the suppliers of lines of motors, bas on their particular form of magnetizing curve, values of resistive volta drop and principal type of application.

## (12) THE USE OF SUCH CURVES IN THE SELECTION MOTORS FOR INDUSTRIAL APPLICATIONS

Curves of the kind given in Figs. 8–13, supplemented in vario ways according to the intended field of use, serve two main p poses. First, they facilitate sufficiently close estimates of performance that can be achieved with any proposed mo without the plotting of run curves. The arithmetic required essentially the evaluation of the appropriate scale facto Secondly, such curves may be used to display the effects,

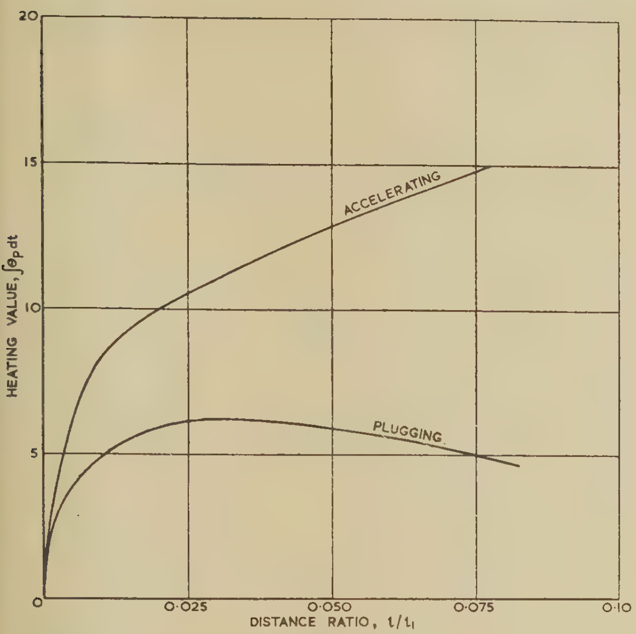


Fig. 12.—Heating values over one run (motoring and braking separately) using values of $\theta_p$ from Fig. 5.

$K_f = 0{\cdot}3.$
$l_1 = u_1^3 W/Vi_1$ feet.
$t_1 = u_1^2 W/Vi_1$ seconds.

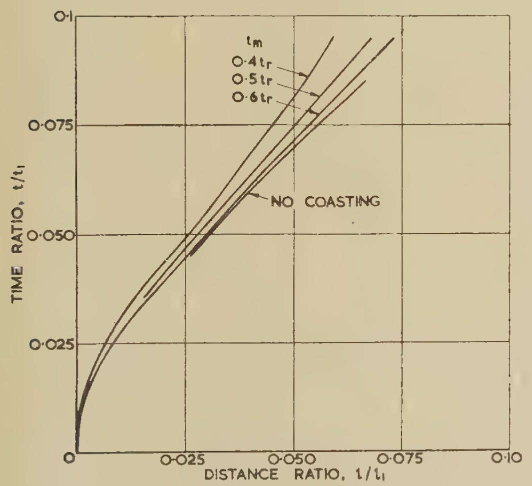


Fig. 13.—Start-to-stop run time ratio as a function of distance ratio for various proportions of coasting time.

$l_1 = u_1^3 W/Vi_1$ feet.
$t_1 = u_1^2 W/Vi_1$ seconds.

operating times, power consumption, motor heating and the like, of the various features and values that are open to choice, such as gear ratio and number of turns of the series winding, accelerating current, proportion of coasting time, and so on.

The preparation of such data is thus a necessary first step towards the formulation of guiding principles for the selection of motors to achieve maximum overall economy. Generalized data of the kind presented, being applicable to a class of duties and a class of motors (frequent-stop services and series motors so far as the present paper is concerned) when associated with the basic economic data, namely the energy tariff, the relation of cost to rating, and the rate of interest, provide a basis for a systematic attempt to specify the combinations to give maximum economy.

It is not the purpose of the paper to analyse the problems of motor application in detail, but a brief indication will be given of how even the limited data presented may immediately be used in practical problems. This is most simply done by considering some particular problem, and for illustration the selection of a long-travel motor for a steelworks crane will be considered.

## (13) THE SELECTION OF A SERIES-WOUND MOTOR FOR THE LONG-TRAVEL MOTION OF A STEELWORKS CRANE

Suppose a crane of given weight and tractive resistance is to be fitted with a motor so that it will adequately and economically perform a required most onerous duty. The most onerous duty consists of the making of some number of runs of a variety of lengths within a stated period. The most onerous intensity of duty should be assessed or averaged over a time that bears some relationship to the time-constant of the motor. This should be a longer time for a large totally-enclosed motor than for a small ventilated motor. It is convenient, however, in all cases, to specify the intensity of duty by the number of runs of various lengths per hour. Such a duty specification may be displayed as a histogram, as shown by the example in Fig. 14. A large proportion of the runs in such duties are inching operations, and for distinction the complete operation of lifting an ingot and returning to the loading point is referred to as a load operation.

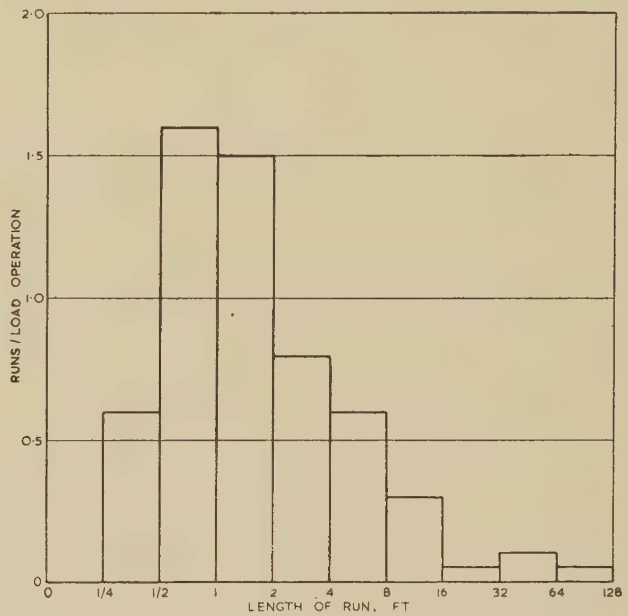


Fig. 14.—Number of runs made per load operation having lengths within various ranges.

Values recorded in service on a crane handling ingots, and assumed to apply in the sample calculation.

It is evident that a motor cannot rationally be selected without some duty being assumed, and it is reasonable and not difficult to make some estimate of the probable contour of such a duty histogram. Measurements of duties in various representative steelworks operations are at present in progress, using automatic counters, with the object of formulating a systematic approach to the specification of motor duties in this manner.

The mere number of runs of various lengths per hour is not a complete specification of the requirements. It is also required that individual runs or operations should be carried out in times that are not too long. A design in which the specified runs could just be made within the hour is not likely to be suitable. There are periods in which demands conflict, and in which waiting

keeps men idle and processes halted. There is therefore required an additional specification that ensures an accepted celerity in individual operations and that will imply as a consequence a proportion of standing time. At present this is a matter of experience, and of the dislike of a 'slow' crane that has its basis perhaps more in dislike of waiting than in economic calculation. An investigation by the B.I.S.R.A.[3] has established as a possible working rule that a crane of the kind in question will be regarded as 'acceptable' if it can move from rest through 8 ft in 3–4 sec. When long runs are involved a minimum speed after a fixed time from rest may also be 'required' for acceptability. This does not necessarily imply justification on strictly economic grounds, since a lower balancing speed, obtained, for example, by using a larger gear ratio, usually implies a higher acceleration for a given starting current, an economy of power and better performance on the shorter runs.

At present a complete elucidation of the economic factors is lacking, and the designer can proceed only by trial, making estimates of the time to move given distances from rest, of the balancing speed, of the temperature rise resulting from the most onerous duty, and of the current consumption, for various combinations of motor, gear ratio, starting currents, amounts of coasting, and possibly of number of turns on the series winding, and so on.

The set of curves, Figs. 8–13, enables these criteria of performance to be estimated for various alternative combinations without prohibitive labour.

In seeking an appropriate combination there are a few guiding principles. The most important is that the starting current at which any proposed motor should be used should be the maximum which temperature rise and other physical limitations permit. It is easily shown that, as progressively larger starting currents are assumed, short runs may be made faster, or the energy consumption, including losses in the starting resistance, is reduced. On the other hand, the motor will usually attain a higher ultimate temperature. Thus there is advantage in increasing the starting current to a value at which either the temperature rise or the current reaches the permissible limit.

A principle of similar generality is that overall economy requires that some coasting should be used. This involves problems of training and supervision of crane drivers, and it may in some cases be thought necessary to cater for the possibility that the crane will be operated without coasting, in spite of the costs so incurred in energy consumption and brake wear and the fact that a motor of larger rating may be required as a consequence of this assumption. Similar considerations about the way the crane is driven arise in respect of the possibility of making runs more slowly and reducing standing time whenever there is no necessity to do otherwise.

With these considerations in mind the procedure for using the curves for such estimates will be largely self-evident. The first step is to locate the 'representative current' on the magnetization curve or on the speed/current curve of the proposed motor, as indicated in Section 2, and to find the value of $K_r$, the factor for resistive drop.

The quantities open to variation are of two kinds, those (like $R$ or $N$) that affect only the scale factors, and those (like the accelerating current or the coasting-time ratio) that imply the use of a different curve.

## (14) AN EXAMPLE OF THE USE OF THE CURVES

The case will be considered in which it is desired to select a suitable motor and gear ratio for the long-travel motion of an overhead crane. As a particular case, suppose the basic values are:

| | |
|---|---|
| Weight of crane and load .. .. .. | 64 tons. |
| Frictional resistance .. .. .. .. | 21 lb/ton. |
| Supply voltage .. .. .. .. .. | 230 volts. |

and let the desired performance be specified by

| | |
|---|---|
| Time to move 8 ft from rest .. .. .. | 4 sec. |
| Speed after 10 sec from rest .. .. .. | 300–400 ft/min. |
| Load operations per hour (e.g. ingots removed) | 35. |
| Lengths of runs, including inching, average per load operation | Distributed as shown in Fig. 14. |

Further, the temperature rise in service is not to exceed 75° C by thermometer as usually measured; this will correspond to an ultimate winding temperature rise of the armature winding (by resistance) of, say, 110° C.

The curves provided make it possible to check the suitability or otherwise of any motor proposed for the application. As an example, let a motor (a particular 35 h.p. motor) be considered for which the following values apply:

| | |
|---|---|
| Representative current (i.e. current at which air-gap ampere-turns are exceeded by 50%) | 101 amp. |
| Speed at 230 volts and at representative current | 575 r.p.m., or 60·2 rad/sec. |
| Ratio ($K_r$) of $ri$ drop to line voltage at representative current | 0·088. |
| Results of a continuous test for temperature rise | 103° C rise at 68 amp and 680 r.p.m. |

### (14.1) Assumption for a Trial Calculation

It will be assumed that the initial accelerating current will be twice the representative current, and that the plugging current will be equal to the accelerating current. It will also be assumed that the motoring current-on time is 60% of the run time in all cases, i.e. that a reasonable proportion of coasting is used.

The form in which the requirement for initial acceleration is given, namely 8 ft from rest in 4 sec, implies an acceleration of 1 ft/sec² if the additional assumption is made that this operation involves only the constant acceleration. This will first be assumed, and checked later to be the case.

### (14.2) Calculation of a Minimum Gear Ratio

The first step is to find the largest value of $R$, the ratio of the linear speed of the load to the angular speed of the motor, that will just permit the motor to give the required initial acceleration and then to check whether the speed reached after 10 sec is sufficient. The factor $R$ involves both the wheel radius $R_\omega$ and the gear ratio $\rho$, since $R = R_\omega/\rho$, so that a required $R$ may be obtained by suitable adjustment of either $R_\omega$ or $\rho$ or both.

The values of $K_f$ and of $\omega_1$ are first required. Since the frictional resistance is 21 lb/ton,

$$K_f = \frac{\text{Retardation due to friction}}{\text{Initial acceleration}} = \frac{21 \times 32\cdot 2}{2240 \times 1\cdot 0} = 0\cdot 3$$

Since $K_r = 0\cdot 088$, and the angular speed $\omega$ at the representative current is 60·2 rad/sec, and by eqn. (7),

$$\omega/\omega_1 = 1 - K_r$$

it follows that $\omega_1 = 66$ rad/sec.

This gives the scale factor for the use of the generalized torque/current curve, Fig. 2, namely

$$M_1 = \frac{Vi_1}{\omega_1} = \frac{230 \times 101}{66} = 352$$

The accelerating current $i/i_1 = 2$, and reading for this value from Fig. 2 gives

$$\frac{M}{M_1} = 1\cdot 79, \text{ so that } M = 1\cdot 79 \times 352 = 630$$

The required linear accelerating force is given by eqn. (19), namely

$$a = \frac{P_a g}{(1 + K_f)W}$$

from which

$$P_a = \frac{1\cdot0 \times 64 \times 2240 \times 1\cdot3}{32\cdot2} = 5800$$

but $P_a R = M$, so $R = \dfrac{630}{5800} = 0\cdot109$ to give the specified acceleration.

### (14.3) The Scale Factors for the Curves

Assuming this value of $R$, the several scale factors are easily calculated, and are:

*For linear speed.* $u_1 = \omega_1 R = 66 \times 0\cdot109 = 7\cdot17$ ft/sec.

*For time.* $t_1 = \dfrac{u_1^2 W}{V i_1} = \dfrac{(7\cdot17)^2 \times 64 \times 2240}{230 \times 101} = 317$ sec.

*For distance.* $l = \dfrac{u_1^3 W}{V i_1} = u_1 t_1 = 7\cdot17 \times 317 = 2275$ ft.

*For quantity of electricity.* $Q = \dfrac{u_1^2 W}{V} = \dfrac{(7\cdot17)^2 \times 64 \times 2240}{230} = 32050$ amp-sec.

based on the measurements on a 25 h.p. motor given in Fig. 5, as follows.

The temperature rise of the 35 h.p. motor at 680 r.p.m. was 103° C. From Fig. 6, the factor for change of speed from 680 to standard speed of 590 r.p.m. is 0·968. Therefore, at $i/i_1 = 68/101 = 0\cdot673$ the potential temperature rise of the 35 h.p. motor is $103/0\cdot968 = 106\cdot4$° C. From Fig. 5, $\theta_p$ for the 25 h.p. motor is 108° C, so that $k_{xy} = 106\cdot4/108 = 0\cdot985$.

It is required to sum the values of $H$ given by Fig. 12 for all the runs in an hour as specified by the histogram, Fig. 14. The term 'load operation' will be used for the complete operation of transporting an ingot, setting it down, and returning ready to pick up another. The term 'run' means a distinct movement of the crane. The number of runs per load operation, from the histogram, is 5·6, and the excess over 2 is due to inching operations. The mean schedule speed for the specified requirement of 35 load operations per hour is given by:

Mean distance travelled per load operation (from histogram) = 24·07 ft.
Mean speed for 35 operations per hour = 0·234 ft/sec.
Mean speed in r.p.m. = 20·5, since $R = 0\cdot109$.

The total heating value of all the runs per load operation is calculated in Table 1 and found to be 20·43.

Allowing for the time scale factor, $t_1 = 317$, the temperature rise (as yet uncorrected since the mean dissipation is not that for the standard speed) is therefore

$$\frac{20\cdot43 \times 317 \times 35}{3600} 0\cdot985 = 62° \text{C}$$

The correction factor required because the schedule speed is 20·5 r.p.m. instead of the standard speed, 590 r.p.m., is 1·58.

The expected temperature rise is thus $62\cdot0 \times 1\cdot58 = 98$° C.

**Table 1**

| Runs of distances between | Mean distance, $l$ | $l/l_1$ | $H$ as read from Fig. 12 | | | Runs per load operation | $H$ per load operation |
|---|---|---|---|---|---|---|---|
| | | | Motoring | Plugging | Total | | |
| ft | ft | | | | | | |
| ¼ and ½ | 0·375 | 0·000165 | 0·32 | 0·32 | 0·64 | 0·6 | 0·384 |
| ½ and 1 | 0·75 | 0·00033 | 0·64 | 0·64 | 1·28 | 1·6 | 2·048 |
| .. | .. | .. | etc. | .. | .. | .. | .. |
| 32 and 64 | 48·0 | 0·0211 | 10·10 | 6·00 | 16·10 | 0·1 | 1·61 |
| 64 and 128 | 96·0 | 0·04292 | 12·30 | 6·10 | 18·40 | 0·05 | 0·92 |
| | | | | | | | 20·43 |

The appropriate curves may now be used to obtain the values that are of interest. Examples are given in Sections 14.4 and 14.5.

### (14.4) Time to Move 8 ft from Rest

From Fig. 9, reading $l/l_1 = 8/2275 = 0\cdot0035$, one obtains $t/t_1 = 0\cdot0126$, so that $t = 0\cdot0126 \times 317 = 4$ sec. This checks exactly with the specified value because only the constant acceleration is involved, as may also be seen from Fig. 8, by noting that the point for $t/t_1 = 0\cdot0126$ falls well within the period of constant acceleration. If this had not proved to be the case, a fresh trial with a slightly larger initial acceleration would have been necessary.

### (14.5) Speed after 10 sec from Rest

In an analogous manner, from Fig. 8 and the curve for $K_f = 0\cdot3$, using $t_1 = 317$ and $u_1 = 7\cdot17$, one finds $u = 384$ ft/min, which is satisfactory.

### (14.6) Rise of Temperature after Prolonged Operation on Duty of the Specified Intensity

The temperature rise of the proposed motor on the specified duty will be calculated using the heating values given by Fig. 12,

### (14.7) Calculation of Current Consumption.

Current consumption may also be a consideration in the choice between alternative motors, and may be calculated immediately as ampere-hours per journey by the use of Fig. 11 and the scale factor $Q_1 = 32050$, as in Table 2.

The value of $\Sigma Q$ per load operation is therefore $0\cdot0983 \times 32050 = 3151\cdot2$ amp-sec. The value of $\Sigma Q$ per hour is $(3151\cdot2 \times 35)/3600 = 30\cdot6$ ampere-hours.

## (15) RELATIONSHIP OF THIS PROCEDURE TO DIMENSIONAL ANALYSIS

The data presented are in the form of curves relating ratios which are in all cases non-dimensional. The non-dimensional groups that may be formed from a given set of variables may be

Table 2

| Runs of distances between | Mean distance, $l$ | $l/l_1$ | Q/Q, per run from Fig. 11 | | | Runs per load operation | $Q/Q_1$ per load operation |
|---|---|---|---|---|---|---|---|
| | | | Motoring | Plugging | Total | | |
| ft | ft | | | | | | |
| ¼ and ½ | 0·375 | 0·000165 | 0·003 | 0·0024 | 0·0054 | 0·6 | 0·00324 |
| ½ and 1 | 0·75 | 0·00033 | 0·0052 | 0·0041 | 0·0093 | 1·6 | 0·01488 |
| . . | . . | . . | etc. | . . | . . | . . | . . |
| 32 and 64 | 48·0 | 0·0211 | 0·042 | 0·024 | 0·066 | 0·1 | 0·0066 |
| 64 and 128 | 96·0 | 0·04292 | 0·0555 | 0·0235 | 0·079 | 0·05 | 0·00395 |
| | | | | | | | 0·09832 |

found by well-known methods described in the literature on dimensional analysis.[5] Dimensional analysis may therefore be used as an alternative means of discovering the product groups that have here been called 'unit quantities'.

The method of deduction used here was preferred to dimensional analysis because the latter, in such cases, provides a variety of alternative possibilities; and geometrical arguments, substantially equivalent to those used in the paper, are in any case necessary to select the forms of presentation of greatest practical usefulness.

## (16) CONCLUSION

The paper has indicated how, by the use of simple notions of similarity of shape of curves, data of considerable generality may be prepared for use in the selection of variable-speed motors. The development of data of this kind, adapted to particular fields of application, not only provides a useful tool for selection of motors in particular cases with security and economy of time, but lays foundations for a future exploration of more general guiding principles in motor application.

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## (18) APPENDIX

Table 3

SCALE FACTORS BY WHICH VARIOUS CO-ORDINATE VALUES ARE RELATED TO THOSE OF A 'MODEL' DRIVE IN WHICH $\Phi_1$, $A_1$, $N$, $K_e$, $V$, $R$ and $W$ ARE EACH UNITY.

| Quantity | Value of scale factor in terms of basic parameters | Value in terms of $\omega_1 = \frac{V}{K_e\Phi}$ and $i_1 = \frac{A_1}{N}$ |
|---|---|---|
| Flux, $\Phi$ . . . . | $\Phi_1$ | $\Phi_1$ } by definition |
| M.M.F., $A$ . . | $A_1$ | $A_1$ } by definition |
| Current, $i$ . . . . | $A_1/N$ | $A_1/N$ |
| Angular speed, $\omega$ . . | $V/K_e\Phi_1$ | $\omega_1$ |
| Torque, $M$ . . | $K_e\Phi_1A_1/N$ | $Vi_1/\omega_1$ |
| Tractive effort, $P$ . . | $K_e\Phi_1A_1/NR$ | $Vi_1/\omega_1R$ |
| Linear speed, $u$ . . | $RV/K_e\Phi_1$ | $R\omega_1$ |
| Time taken, $t$ . . | $R^2VNW/K_e^2\Phi_1^2A_1$ | $R^2\omega_1^2W/Vi_1$ |
| Distance, $l$ . . | $R^3V^2NW/K_e^3\Phi_1^3A_1$ | $R^3\omega_1^3W/Vi_1 = u_1t_1$ |
| Ampere-seconds, $Q$ | $R^2VW/K_e^2\Phi_1^2$ | $R^2\omega_1^2W/V = i_1t_1$ |
| Energy, $E = QV$ . . | $R^2V^2W/K_e^2\Phi_1^2$ | $R^2\omega_1^2W = i_1t_1V$ |
| Heating value, $H$ . . | as for $t$ | |

In all cases the scale factors are expressed by the same symbol as the quantity, with suffix unity added.
The mass of the crane has been expressed in terms of its weight $W$ because it is habitually referred to as such. The factor $1/g$ has been taken into account in the construction of the curves shown in Figs. 8–13.

# DISCUSSION ON THE ABOVE THREE PAPERS BEFORE THE UTILIZATION SECTION, 19TH APRIL, 1956

**Mr. C. C. Inglis:** May we remind ourselves of the basic phenomena of heat: its generation is a volume phenomenon and its dissipation a surface phenomenon, the one varying with the cube of the dimension and the other with the square. Therefore, the bigger the machine the more difficult it becomes.

I wonder whether the same strict analytical approach has been made to heat problems in electrical machinery as has been made on the electrical side. For instance, it is common to flux-plot fringe areas in the magnetic circuit. Is a similar flux-plotting technique used for the establishment of a temperature gradient or a hot spot or any thermal phenomenon within the body of the machine?

Machines of the traction type, when producing greater output than their continuous rating, are essentially energy producing machines, and must be regarded as such, and not in terms of their horse-power.

Some years ago various supply companies asked their customers to check their a.c. motors to see whether they were overrated and therefore working at a bad power factor. Surveys of this kind showed in a number of cases that machines were working at half or quarter load. Rearrangement of loads, etc. resulted in an improvement in power factor, to the users' financial advantage. A parallel case might be made regarding temperature. I wonder how many of those using this type of motor take a reasonably regular check on temperature. After all, it is the temperature of the motor that matters, not the amount of current it takes, and it is temperature that decides its life. Hence it is of first-order importance in economics. That leads directly to the conception that the ammeter is not of primary importance.

What we want to aim at is an accurate temperature indicator which has a memory and which knows what has gone before and how much more can be got out of the machine until it reaches a dangerous temperature.

To deal more directly with the papers, there runs through them the concept of the electrical analogue of the thermal network

One of the illustrations shows temperature and its electrical analogue, potential. Thermal conductance is taken as the analogue of resistance, and heat flow presumably as the analogue of current. That is all right so long as you stick to those three simple quantities; but if you go one step further and carry the analogy to energy considerations, you get into trouble. Whereas in the electrical case $I^2R$ is the power loss when current $I$ flows through resistance $R$, the square of the heat multiplied by the conductance is nothing like the power loss.

I do not see anywhere in the paper the concept of energy, and I am wondering whether a simpler approach might not be possible through a more accurate analogue. A paper by Bosworth in the *Philosophical Magazine*, November, 1946, called 'The Thermal Ohm, Farad and Henry' is of interest. The authors state that heat flow is not analogous to current, whereas potential might be analogous to temperature. They point out that a closer analogy is that of electrical charge with entropy; they show that this holds for energy considerations; but they get into a little trouble when they come to thermal conductance, and have to go to hydrodynamics to get out of the difficulty.

A body has a capacitance of one thermal farad if an amount of entropy of one joule per deg. C added to that body raises the temperature by 1°C. For 1 cm$^3$ of copper at 20°C the thermal capacitance amounts to 11·6 thermal mF. Obviously, it is a very nice demonstration that heat is a slow-moving phenomenon and electricity a fast-moving one.

**Mr. A. Asbury:** In the third paper the authors mention that their method has been evolved to reduce to manageable proportions the calculation necessary to select the size of motor. A digital computer would also handle such calculations, which could then be made without assumptions of affinity, and the data for particular sizes of motors would be used for successive calculations for the specified conditions of the application.

This would appear to suit parameters such as friction torque which remain constant better than the authors' method in which $K_f$, the relation of friction torque to accelerating torque, must be constant to give the maximum simplification. Even so, the systematic approach suggested by the authors would help to reduce the labour involved when using a computer.

From the last part of Section 14.6, it is clear that the correction factor for speed may have a large effect on the final answer. With very little more calculation, Table 1 could include a mean speed for each of the run distances. This would allow individual corrections to the values of $H$ per load operation and so give an answer which would be more accurate.

It will also be realized that the size of motor is frequently determined by considerations other than heating. It may, for instance, in the screwdown for a hot reversing mill be determined by the torque necessary to screw down when there is a piece between the rollers.

**Mr. J. A. Broughall:** The first paper seems to assume that the cooling medium is a neutral; is this always true? For example, in the case of the turbo-alternator the cooling air is far from being a neutral, and the change from air to hydrogen cooling has entirely altered the whole thermal conception of the machine. It is in fact one of the reasons why the large modern machines can be made with small windage loss, and so allow the dissipation of the heat generated in them.

For the particular example that has been examined it may not matter, but perhaps the point is worth bearing in mind. The conception of these papers is fundamentally useful, but if as in this case a proposition is being enunciated as fundamentally true it ought to apply to far more than mill motors.

If one does not treat the cooling medium as a neutral, one cannot enunciate the fundamental principles in such simple terms as the analogue shown in Fig. 2.

The subject is important to me and to many other people because we are about to embark in British Railways on a system of electrification using alternating current. We are about to develop all kinds of machines which have not been produced in this country before, and it is of the greatest possible importance that they should be developed so as to be efficient, light, cheap and safe.

We are doing this at what seems to be a very fortunate time. New methods offer a promise of using machines at much higher temperatures. We ought to use new insulating materials, in order to take advantage of the possibility of running these machines at higher temperatures. But to do so with safety we need to know all there is to know about these machines before we start putting them into service. Now, of course, we have to put them into service before we know anything about them. That is why this analytical approach is important, and it applies to far more than the traction motor. It applies, in the case of a.c. traction, equally to the transformer, perhaps even to the switchgear, but certainly to the transformer and to all the auxiliaries.

These papers should be quite fascinating to a traction engineer because the very breath of his life is a variable load, chopping and changing all the time. But it will be necessary, I think, to distinguish between suburban loads and main-line loads, which will be entirely different. In the one specific reference to traction in the papers (Tustin and Bates, Section 4), it is said that an electric train accelerates, for example, at about 2 m.p.h. per second and is only on the grids for about 6 sec. Ordinary suburban electrification involves being on the grids for 20/30 sec up to a speed of the order of about 25 m.p.h. I do not think, therefore, that the statement in the paper that the increases in temperature during these short rushes of current during accelerating are substantially negligible is tenable in most suburban electrifications.

I have some doubt whether the fundamental proposition that you can get the right answer about temperature rise by doing certain tests standing and others at full current is really tenable. I am sure the authors will agree that that proposition should be put to the proof on more than one type of machine. If it is true for one it should be true for others. Therefore one should encourage them to put it to the proof as soon as possible.

**Dr. E. Hughes:** The authors have brought order out of chaos in dealing with the complicated subject of temperature rise in electrical machines. There are one or two points concerning the method of temperature measurement on which I am not clear; for instance, there are references to the temperature rise of the armature core and of the commutator, but no information is given as to how these measurements were made. Also, there is no reference to any measurement of temperature rise by thermometer. It would have been useful to know how the temperatures actually measured compared with those determined by methods used in commercial testing.

The reciprocity theorem is not applicable to some cases of heating; for example, in a protected-type d.c. machine the loss in the field winding has no effect on the temperature rise of the armature, whereas the armature loss may have a considerable effect on that of the field winding.

If the reciprocals of the heating coefficient for the armature winding and core are plotted against speed, the graphs are approximately linear over a large range of speed, and this relationship might be found to be a more convenient form than that given in these papers.

Before much of the information given in these papers can be applied, it is necessary to determine the actual losses under load conditions. This was a matter I realized very soon after commencing a series of experiments on the heating of electrical

machines many years ago, and led to an investigation of the effect of flux distortion on the magnitude of the iron loss.*

**Mr. W. Hill:** Supposing a steelworks requires a short-time-rated crane motor and invites quotations for a particular output. This output would usually be specified by the mechanical engineer in charge. The motor manufacturers concerned are unlikely to be able to spare the time to go into the lengthy calculations required to check this output, especially as they are not really interested in reducing the size of a machine they are going to build. They have every incentive to supply a large machine. Nowadays, the customer also is far too busy to allow the lengthy measurements, which the authors show to be necessary, to be carried out in his own works. So the usual British Standard half-hour or one-hour-rated machine is ordered.

The Swiss standards are different. They make a distinction between short-time and intermittent ratings. Would it not be sensible to try first to introduce some such distinction between, say, duty-cycle machines and intermittent-rated machines into British Standards? Further refinements on the authors' lines may then follow in time.

**Mr. P. Wegner:** I am a mathematician, and am trying to formulate a procedure whereby the whole problem of design can be dealt with as tidily as possible on an automatic computer.

It is well known that the use of an automatic computer, in dealing with this problem, demands an approach fundamentally different from that required when no computer is available and it is necessary to minimize the work of calculation. With a computer the length of the calculation presents no difficulty, but the design procedure must be rigidly fixed, and it must be quite certain that every step is logically defined.

In the engineering company for which I work, I have found that the design procedure is often based on *ad hoc* methods, which are not always logically justified. Such empirical procedures lessen the amount of calculation involved, but it is admitted that they are often based on intuition. It would appear that this approach must be modified if electrical machines are to be designed with the aid of an automatic computer.

I am grateful to Prof. Tustin and his co-authors for dealing with the problem of design in a general way, and I hope that this kind of work will eventually lead to a procedure whereby the most efficient machine can be designed from a given initial specification, merely by feeding that specification into an electronic computer and obtaining as output all the required design parameters.

**Mr. C. F. R. Fielden:** I think I detect a current in the discussion which suggests that people are drifting away from the original purpose of this work. The steelworks auxiliary motor is a standardized machine, and ten frames have been developed as standardized motors. The object is not—as perhaps one speaker thought—to provide data to design a different motor for each job: it is work designed to give a correct assessment of a duty, so that one of these standardized frames can be selected and used to the best advantage.

One of the things we are always up against—speaking as one of the people engaged in selling these machines—is that people will often insist on having a machine which we know to be too big, and which will waste energy during its acceleration periods. On severe duties this may lead to a difficult heating problem. Within my own experience it has been possible to achieve better results by reducing the size of the motor installed on the long travel motion of a crane.

The point I want to make is that the object of the exercise is economy in the use of standardized mill motors. If, now, the concepts put forward by the authors can be followed up by work in the field—and I understand this is being done—to give an assessment of the duty cycles of the different applications of these motors, then indeed we shall be getting somewhere.

**Mr. J. H. Messenger:** In the third paper it is stated that a much more extensive set of data curves is needed for practical use. As a motor designer I should be responsible for providing such curves, and I should like to know how much more data is envisaged by the authors.

Secondly, for many applications the frame size of the motor is based on maximum torque, and in the case of a d.c. motor the maximum torque is fixed by the size of the machine and its ability to commutate the peak currents. If we wish to increase the temperature rise and retain the overload capacity, we can only do this by reducing the amount of copper in the machine, not by reducing the frame size. If we reduce the amount of copper we must have a different conception of the standard works test, because, although the maximum permissible torque will be unchanged, the rating to give a temperature rise of 75° C after one hour will be greatly reduced.

**Dr. L. N. Bramley:** In response to the request of the last speaker for more data on the heating value curves, I, as a representative of the British Iron and Steel Research Association, would like to say that it is our intention to test the standard range of mill-type motors on the lines indicated and to publish the results for the benefit of the steel industry and other industries using mill-type motors.

## THE AUTHORS' REPLY TO THE ABOVE DISCUSSION

**Professor H. Tustin, Dr. J. J. Bates, Messrs. D. F. Nettell and R. Solt** (*in reply*): The discussion shows the desirability of making a clear distinction between two problems relating to the heating of machines. The papers are principally concerned with the problem of predicting how hot a motor of a particular kind and size will become when it is used on a given duty. The estimate is based directly on the results of tests on nominally identical motors, and avoids any necessity to have knowledge of the losses or the thermal conductances. These are referred to in establishing and validating the method, but are not involved in its use.

The wider problem, to which some contribution may be made by the papers, but which is not their primary topic, is the problem of designing a motor for a given rating or duty, or, when a design exists in the form of drawings, of predicting how hot it will get. This problem involves, for its radical solution, the ability to calculate the thermal conductances and all the losses in detail from the dimensions.

Several of the comments made on the papers are of the nature of contributions to this wider problem, as in Mr. Inglis's reference to the value of graphical solutions for conduction problems by methods analogous to flux plotting, and in Dr. Hughes's reference to the difficulty of determining losses under load conditions.

Mr. Broughall's appreciation of the potential usefulness of the ideas in the papers in present developments in electric traction is most welcome. This field of application, in which so large an expenditure on electric motors and associated transformers and other apparatus is about to be made, includes some cases in which the procedures described are applicable, namely the class of sustained short-run services. In main-line traction, where the duration of periods of high load may be comparable with the thermal time-constants, the maximum temperature attained may considerably exceed the mean temperature, but the effects of thermal capacitance, as possibly limiting the maximum temperature, must be taken into account. Further work is in progress

* 'Iron Losses in D.C. Machines', *Journal I.E.E.*, 1924, 63, p. 35.

aiming at adapting the same simple approach to this rather more complex problem. Mr. Broughall rightly calls attention to the fact that the basic condition for the direct application of the procedures described is that the periods of high load, as in accelerating, should be short. He questions whether periods of 20–30 sec might be too long, but even neglecting heat dissipation altogether, the temperature fluctuations about the mean in 30 sec could be only of the order of $\pm 2\frac{1}{4}°$. Allowing for dissipation they would be even less, and a simple means for estimating the maximum temperature is given in Section 15.1. He is also justified in noting that one cannot assume that heat dissipation when standing is related in a simple way to that when running. We agree that the dissipation at standstill should be investigated in every case, and much more experimental evidence is necessary before any generalization can be made.

Both Mr. Asbury and Mr. Wegner refer to the use of digital computers in such investigations. Here again the possibilities opened up to designers are great, but they do not make it unnecessary for the user to be provided with adequate data, in a simple form, about the capabilities of machines, and simple physical concepts to guide him in their application.

Mr. Messenger asks how many more data (derived from run curves) are required for practical use. For simplicity of presentation the number of curves given in the paper was kept to a minimum. For example, in Fig. 11 only one curve is given for current consumption during motoring as a function of distance. This was for a particular form of magnetization curve, with particular values of resistive-drop ratio (0·088), the ratio of motoring time to running time (0·6), the ratio of frictional retardation to starting acceleration (0·3), and for a starting current of twice the 'representative' current, i.e. for particular values of five variables. Strictly, every different combination of these five variables results in a distinct curve, but some of them, over the range of possible variations, have little effect. To cater for combinations of only two each of five variables would require $2^5$, i.e. 32 distinct curves. Exploration and careful consideration of the practical needs are therefore required in planning the preparation of such data. We believe that the analysis given in the paper provides the most compact representation of motor performance that is theoretically exact. Further economy in presentation must be a matter of practical compromise and approximation according to the application in question.

Mr. Inglis refers to other analogies between thermal and electrical quantities. Interesting as these are, there would be no advantage in using them in problems of the kind under consideration, in which the analogy of heat flow with current and heat storage with charge is self-evident, exact and adequate.

Dr. Hughes asks for details of the temperature measurements. These were by embedded resistance-thermometers in the various parts of the armature; holes were drilled at each end of the core just under the slots; the stampings were then separated, the burrs were removed and the core rebuilt. The temperature-sensitive resistors were fitted in these holes. In the commutator the resistors were fitted in a small recess cut at the bottom of the risers, and in the end windings they were placed between the winding layers. The resistor in the part of the winding in the slots is described in detail in the papers.

Mr. Asbury makes the useful suggestion that the correction for speed should be made on the values of $H$ for each separate run, according to the mean speed for that run. In principle we agree, but we would make this improvement rather differently: the most significant need is to include the effect of standstill time correctly; the factor for correction of temperature for speed has been given as a multiplying factor, i.e. as a ratio of equivalent thermal resistances.

If one proposes to calculate the temperature rise when the thermal resistance is $K_{s1}R$ for a period $t_1$, $K_{s2}R$ for a period $t_2$ and so on, where $R$ is a thermal resistance for the standard speed, it is easily seen that the correction factor is strictly the reciprocal of $[(t_1/K_{s1}) + (t_2/K_{s2}) + \ldots]$, i.e. strictly one should average the conductances, and so the reciprocals of the factors $K_s$. It might thus be convenient to record the reciprocal of the temperature-correction factors, $1/K_s$, instead of $K_s$, since these values may more correctly be averaged over time. If the values of $K_s$ are never very different from unity, this point is not significant.

Mr. Hill aptly explains how it comes about that motors are usually ordered on a basis of the standard half-hour or one-hour rating. He appears to suggest the use of the Continental method of specifying on-off time ratios at various ratings for standard temperature rises. What we are advocating amounts to the same thing, with the significant improvement that the data from the intermittent tests should be recorded as the temperature rise multiplied by the ratio of total time/'on' time (i.e. the potential temperature rise), which permits averaging; and that sufficient tests should be made to permit continuous curves to be drawn. Certainly one could proceed in stages and complete the records as opportunity occurred, but the important need is to establish the habit of recording the results in this more useful form.

# DISCUSSION ON

## 'EQUIPMENT OF INSTRUMENTAL ACCURACY FOR RECORDING AND REPRODUCTION OF ELECTRICAL SIGNALS, USING CINEMATOGRAPHIC FILM'*

### NORTH-EASTERN RADIO AND MEASUREMENTS GROUP, AT NEWCASTLE UPON TYNE, 19TH MARCH, 1956

**Mr. G. H. Hickling:** The author has made clear his preference for the variable-area system over the alternative one which is described first. To anyone who has not made a close study of the subject, it would appear a hopelessly difficult task to obtain any accuracy at all from the variable-density scheme, bearing in mind the difficulty of obtaining suitable controllable light sources, the variability of photographic materials and processing, the effect of film-speed variations, etc. (One only gathers incidentally that it is necessary, not only to process the original negative, but also to make a positive print.) The description in Section 11.4 of the very elaborate processing technique, and of other 'insidious troubles' in the photocell feedback loop and elsewhere, only serves to substantiate the doubts regarding the usefulness of this process for general measurement work. I am left wondering, therefore, why this particular technique was not omitted from the paper altogether.

Evidently, by the careful trial-and-error adjustment of the processing method, a reasonable standard of accuracy was achieved in the development trials of the system; but would such accuracy be maintained in routine use of the method—and how could it be checked?

The superior performance of the variable-area process is again exactly what would be expected from a knowledge of the performance obtainable from Duddell oscillographs. The advantage, mentioned in passing, of a system relying 'on a dimensional effect on the photographic film rather than on a transmission effect, which requires precision processing' appears to be quite overriding. Even so, this advantage is partly lost in the reproduction process, as shown by the necessity for employing a reference channel to compensate for light-intensity variations. The adoption of a subtraction process for this purpose, rather than a division one, is an instance of the sacrifice of performance to attain simplicity—yet still without avoiding a fair measure of complication. The provision of 2-channel recording is mentioned as a specific requirement. I wonder, however, whether a greater number of channels would not have been desired, had this proved feasible.

Reviewing the systems described as a whole, the thesis of the paper appears to be that, as compared with other recording methods available—e.g. magnetic tape and photographic recording on the cathode-ray oscillograph—they provide superior accuracy and, at the same time, afford a facility, in the possibility of playback, not afforded by the cathode-ray-oscillograph technique. However, neither contention is necessarily true. Good flat-screen cathode-ray tubes are now available on which a deflection accuracy and linearity of $\frac{1}{2}\%$ can readily be obtained, and which, with correct operation, give a correspondingly fine trace over a wide range of writing speeds.

The requirement of playback from a normal moving-film cathode-ray-oscillograph trace, recorded with a zero line, could, I believe, be met by means of an equipment arranged to scan the film at right angles to its direction of motion, with a moving light spot and photocell. This would result in two voltage pulses per scan (for a single recording track), the time separation of which would be a measure of the recorded voltage. Compensation for lateral movement of the film would be automatic. Several time-interval measuring circuits exist by means of which this information could either be reconverted to a voltage amplitude, or, if required, made available in digital form. Films carrying multiple recording could also be dealt with, by using an electronic time-gating method, provided that they did not actually overlap.

**Mr. H. McG. Ross** (*in reply*): When the work described was begun, a variable-density system had been made to perform tolerably well in the laboratory, and it was desired to continue its development. The whole project was regarded partly as an experiment to determine the relative merits of the variable-density and the variable-area systems, and, in fact, the superiority of the latter did not appear conclusively until a late stage of the work.

The basic requirement of this equipment was to give playback of the recorded signals, in order to facilitate the analysis of the data obtained during complex experiments. Accordingly, every aspect of the recording process was influenced by the requirement for playback. This distinguishes the work from others, before and since, in which mere recording at high accuracy was sufficient.

In any recording and playback system the overall errors seem to be compounded of a great number of minor effects, each of which on its own is usually small enough to pass unnoticed. The use of the new high-accuracy cathode-ray tubes, and a playback system based on measurements of time intervals, might be promising. But it is felt that the true performance could only be assessed by making and testing the system, in order to determine whether the minor errors build up.

The variable-area system described in the paper could probably be easily developed to carry three channels; any further increase in the number of channels would probably result in lower accuracy.

* Ross, H. McG.: Paper No. 1696 M, August, 1954 (see **102 B**, p. 323).

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Paper No. 2177 S
Sept. 1956

# A DUAL-COMPARATOR MHO-TYPE DISTANCE RELAY UTILIZING TRANSISTORS

By COLIN ADAMSON, M.Sc.(Eng.), Associate Member, and
L. M. WEDEPOHL, B.Sc.(Eng.), Graduate.

(*The paper was first received* 27*th February, and in revised form* 17*th May*, 1956. *It was published separately in September*, 1956.)

SUMMARY

The paper describes further work on a mho-type relay using transistors, which depends for its operation on direct phase-comparison. The relay is rendered free from transient overreach by the use of a dual comparator circuit configuration. Relay accuracy is discussed and the methods which have been employed to improve the accuracy of the relay during operation without increasing its sensitivity are described.

The tests applied to the relay and the reasons for their adoption are given briefly. Accuracy/range charts and dynamic mho-curves are presented for the relay connected as a phase-fault element with normal polarization, with polarization from an unfaulted phase and with the use of memory.

It is concluded that the relay is a marked improvement over prototypes described earlier by the authors, particularly in respect of accuracy and absence of transient overreach. Speed of operation is discussed, with particular regard to the use of an alternative criterion of operation in which only one of the two comparators must detect a fault before a tripping signal is produced.

Probable improvements in the relay design with a view to economy in the number of transistors are considered. No transistor failures have been experienced.

## (1) INTRODUCTION

Electronic relays have a number of advantages and disadvantages when compared with electromagnetic relays. In general, electronic relays using thermionic tubes have not met with much favour in the electricity supply industry because of their requirements in heater and anode supplies, although their superiority in respect of negligible inertia, low drain on current and voltage transformers and speed of response has been recognized.[1] Junction transistors overcome the limitations of thermionic valves so far as h.t. and heater requirements are concerned whilst affording all their advantages; no heater supplies are needed and the h.t. demands are modest with regard to both voltage level and power.

A previous paper by the authors[2] has described how this situation led to the initiation of a research project at the Manchester College of Science and Technology, in which potentialities of the junction transistor were investigated for certain protective-gear applications. This work resulted in the design and testing of two mho-type distance relays, one of which was subsequently discarded, the other meeting with a moderate degree of success. It was shown that these relays had limitations which made them unacceptable in practice; the limitations were due not to the transistors but to the relay circuits. The major difficulty in the case of the second relay was faulty operation due to transient overreach in the presence of d.c. transient components in the outputs of the transformers supplying the relay. Proposals were made with a view to overcoming these difficulties and one of them has now been put into effect. Apart from this, further research and development has now been carried out, and the purpose of the paper is to describe the physical arrangement, testing and test results of an improved transistor mho-type distance relay, in which all the limitations of the two earlier relays have been overcome.

## (2) IMPROVEMENTS NECESSARY TO EARLIER RELAYS

The relays described in the previous paper[2] both made use of phase-comparison principles. The first relay[2,3] used a pulse technique and was found to be unacceptable because random pulses tended to cause spurious operation; for this reason, further research work on this type has been postponed. The second relay[2,4] used a direct-phase-comparison principle; it suffered from the disadvantage of excessive overreach in the presence of d.c. transient components of fault voltage and current. A further disadvantage was the falling-off of performance with decreasing fault current, due to the presence of static bias in the relay input circuit. It was thus necessary to devise means for overcoming both of these drawbacks.

The principle of operation of the direct-phase-comparison relay, which has been described fully in the previous paper,[2] is fundamental to the discussion which follows, and is briefly summarized below.

### (2.1) Basic Direct-Phase-Comparator Principle

Fig. 1 is a block diagram of the basic direct phase-comparator. $V_1$ and $V_2$ are voltages derived from system voltage and current.


**Fig. 1.**—Block diagram of basic phase comparator.

Fig. 2 shows the output waveforms. The coincidence circuit is such that it produces a rectangular output voltage, the duration of which is the period during which the voltages $V_1$ and $V_2$ are both instantaneously positive. The integrating circuit converts this voltage block into a triangular waveform, the peak amplitude of which is a measure of the duration of coincidence. The level detector is a circuit which produces an output voltage pulse when its input voltage attains a predetermined level.

The circuit represented by the block diagram is adjusted in such a way that, for a phase displacement of 90° (electrical) between the voltages $V_1$ and $V_2$, the peak voltage attained in the integrating circuit is just sufficient to operate the level detector, and for phase displacements greater than 90° it is insufficient to cause operation. It is important to note that measurement only takes place when the two voltages are simultaneously positive, the interval when both voltages are negative being discarded. If, in a similar comparator, the input voltages are $-V_1$ and $-V_2$, an output voltage block will be produced over the interval when these are both instantaneously positive, i.e. when $V_1$ and $V_2$ are both instantaneously negative. Provided that these signals are

Mr. Adamson is at the Power Systems Laboratory, Department of Electrical Engineering, Faculty of Technology, University of Manchester.
Mr. Wedepohl is with the Electricity Supply Commission of South Africa, and is at present at the Power Systems Laboratory, University of Manchester.

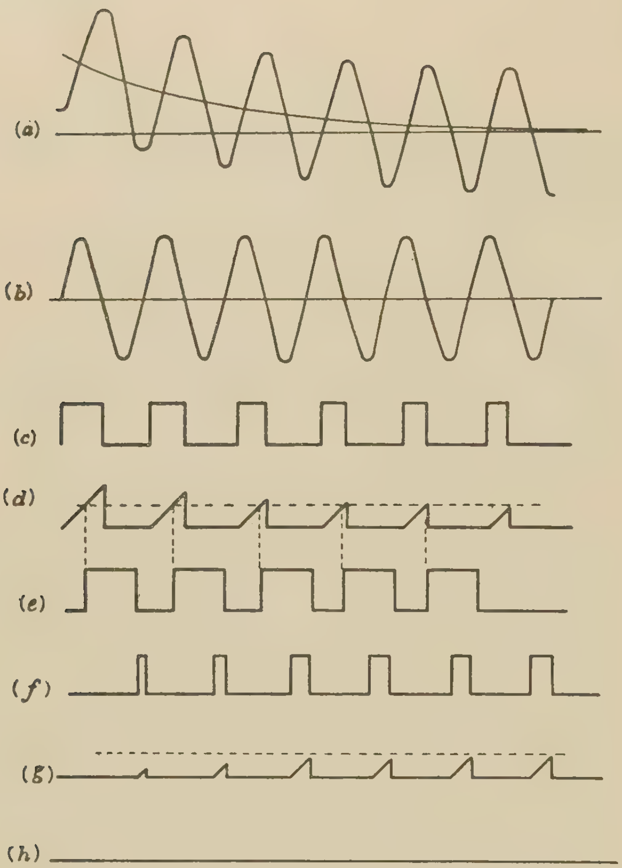


Fig. 2.—Comparator waveforms.

(a) $V_1$.
(b) $V_2$.
(c) First coincidence circuit output.
(d) First integrating circuit output.
(e) First level-detector output.
(f) Second coincidence circuit output.
(g) Second integrating circuit output.
(h) Second level detector output.

sinusoidal, the only difference between the two arrangements is that measurement takes place on alternate half-cycles.

The presence of a d.c. transient component in either $V_1$ or $V_2$ modifies the circuit behaviour, since the voltage wave is no longer symmetrical about the zero position. If the d.c. component is positive the comparator will indicate a longer duration of coincidence, and the converse will hold true for a negative d.c. component. Fig. 2 shows voltages $V_1$ and $V_2$, with a d.c. component present in $V_2$, a sufficient number of cycles being indicated for steady-state conditions to be attained. The waveforms of the outputs from the coincidence, integrator and level-detector circuits are also shown, the first group relating to input voltages $V_1$ and $V_2$, and the second group to a comparator supplied with voltages $-V_1$ and $-V_2$. Superimposed on the integrating-circuit waveform, in each case, is a horizontal line representing the level-detector operating voltage. These waveforms show that, for a positive d.c. component, the comparator tends to give a false indication; this is the mechanism of transient overreach encountered in previous relays. In the comparator supplied with voltages $-V_1$ and $-V_2$, however, the d.c. component is negative and there is no tendency to give an incorrect indication.

### (2.2) Circuit Arrangement for eliminating Transient Overreach

It has been shown in Section 2.1 that only positive d.c. transients in a phase comparator tend to cause false operation.

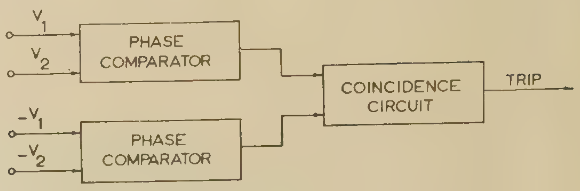


Fig. 3.—Block diagram of dual phase comparator.

This forms the basis of a relay which is shown in block form in Fig. 3. In this circuit, two identical phase-comparators are energized with voltages $V_1$, $V_2$ and $-V_1$, $-V_2$, respectively. The two comparators have identical characteristics provided that $V_1$ and $V_2$ are sinusoidal, but measurement takes place on alternate half-cycles. The outputs from the two comparators are fed to a coincidence circuit, arranged so that an output signal is generated only when both comparators indicate a fault simultaneously. Since a tendency to false operation occurs only in the comparator in which the d.c. component is positive, a relay based on this configuration is free from transient overreach; the effect of both negative and positive d.c. components is quantitatively analysed in Section 9.1.

### (2.3) Compensation for Input Bias

It has been shown in the previous paper[2] that a phase comparator has a mho characteristic if the voltages $V_1$ and $V_2$ are of the form:

$$V_1 = I_L Z_R - V_L$$
$$V_2 = V_L$$

where $V_L = V_L\angle 0$ = System fault voltage, referred to the voltage-transformer secondaries.

$I_L = I_L\angle -\phi$ = System fault current, referred to the current-transformer secondaries.

$Z_R = Z_R\angle \theta$ = A transfer impedance usually obtained through the use of a transformer reactor.

$Z_L = Z_L\angle \phi = V_L/I_L$ = Impedance presented to the relay input terminals.

Normal adjustment is such that $\theta$ is made equal to $\phi$ under line fault conditions, so that $I_L Z_R$ is in phase with $V_L$. Three conditions arise:

(a) $I_L Z_R \geqslant V_L$, i.e. $Z_L \leqslant Z_R$. In this case $V_1$ is in phase with $V_2$ and the phase comparator indicates a fault.

(b) $I_L Z_R < V_L$, i.e. $Z_L > Z_R$. In this case $V_1$ and $V_2$ are in anti-phase and the comparator gives no fault indication.

(c) $I_L Z_R$ is in antiphase to $V_L$, and $V_1$ and $V_2$ are in anti-phase. This condition arises for a fault in the reverse direction and the comparator gives no fault indication.

With this adjustment the mho-type relay behaves as a directional impedance element; $Z_R$ is known as the nominal setting and corresponds to the length of protected feeder. It is customary to refer to $V_1$ as the measuring voltage, the components $I_L Z_R$ and $V_L$ being referred to as the 'operate' and 'restraint' voltages, respectively. $V_2$ is referred to as the polarizing voltage.

In practice, the condition $I_L Z_R \geqslant V_L$ is insufficient to cause relay operation; it is necessary for the operate voltage to overcome both the restraint voltage and the relay input-bias voltage, $V_b$. Thus the criterion for operation becomes

$$I_L Z_R \geqslant V_L + V_b \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Rearranging,

$$\left.\begin{aligned} 1 &\geqslant \frac{V_L}{I_L Z_R} + \frac{V_b}{I_L Z_R} \\ 1 &\geqslant \frac{Z_L}{Z_R} + \frac{V_b}{I_L Z_R} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From eqn. (2) it may be observed that the relay performance approaches the ideal as the term $V_b/I_LZ_R$ tends towards zero. There are two possible ways of arranging this. First, $V_b$ may be reduced as far as possible; this has its counterpart in an electromagnetic relay in which friction is reduced to a minimum and the moving parts are floating, thus requiring very small power for operation. Very high sensitivity in a relay is undesirable since it may lead to instability. Secondly, a more satisfactory approach is to arrange for the term $V_b/I_LZ_R$ to be reduced to zero, dynamically, during the time that the relay is energized. This is achieved by deriving from $V_b$ an equal and opposite voltage from the fault voltage $V_L$ appearing at the relay input terminals. In this way the sensitivity during operation is increased without any change in the static sensitivity as defined by $V_b$.

A simple circuit for deriving the compensating voltage from the fault voltage is shown in Fig. 4. In the electronic phase

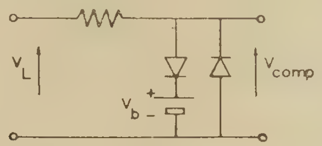


Fig. 4.—Compensating circuit.

comparator, the compensating voltage need not be a steady direct voltage, since measurement can take place only on the positive half-cycle of the polarizing voltage. The compensating voltage is in the form of square waves of amplitude equal to the bias voltage on the measuring half-cycle, and is clipped to zero voltage on the negative half-cycle of polarizing voltage. Furthermore, in the mho-type relay it is necessary to apply compensating voltage only on the measuring side, since it is the phase angle, and not the amplitude, of the polarizing voltage which is important.

## (3) THE COMPLETE RELAY

### (3.1) Block Diagram of the Complete Relay

A block diagram of the complete relay is shown in Fig. 5. Voltage and current are applied to a measuring circuit in order to derive the measuring voltage $(Z_RI_L - V_L)$. $V_L$ is also used to provide polarizing and compensating voltage as shown. All functions are duplicated to provide the dual-comparator facility of measurement on both half-cycles of the input voltages.

### (3.2) Circuit Diagram of the Complete Relay

The complete relay circuit is shown in Fig. 6; one comparator only is shown, since they are identical. It is unnecessary to describe the comparator and trip circuits since the arrangements are very similar to those of the original direct phase-comparator, which has been described previously.[2] The following differences should be noted, however. The common emitter connection on transistors T1 and T2 is returned direct to the +2-volt supply. In the earlier relay, this connection was made through a voltage divider and provided a greater input sensitivity; to compensate for this 2-volt bias, the restraint voltage is applied to a diode limiter circuit, as described above. T3 is the decoupling transistor; T4 and T5 form the level-detector circuit as before.

A further difference is to be found in the second coincidence circuit containing transistors T6 and T7. The operation is as follows. Normally, transistor T5, the output transistor of the level detector, is conducting, as is the corresponding transistor in comparator No. 2, and thus the input voltages to transistors T6 and T7 are both +2 volts. These two transistors are in the earthed-base configuration and under these conditions are both conducting, the output potential at the common collector connection being positive. It is necessary for both these transistors to be cut off before any change in output potential takes place. Since the level detectors associated with the two comparators operate on alternate half-cycles, it is necessary to provide a signal of adequate duration for coincidence to take place; this is ensured by making the pulse duration from each level detector 15 millisec. As in the previous circuit,[2] transistor T8 is a pulse rectifier and T9 is the final tripping transistor for operating the slave relay.

## (4) PERFORMANCE TESTS APPLIED TO THE RELAY

As with earlier relay models, the dual-comparator mho-type relay was tested under exacting conditions on a test bench equipped for the investigation and testing of distance protective-gear schemes. A schematic of one phase of this apparatus is shown in Fig. 7; line impedance was variable in magnitude, the $X/R$ ratio being constant, the source impedance was variable in steps, the $X/R$ ratio being nominally 30 but varying somewhat with the magnitude setting.

### (4.1) Accuracy/Range Charts

#### (4.1.1) Basis of Accuracy/Range Charts.

Various methods of presenting the dynamic characteristics of distance relays have been developed in the past. These include:

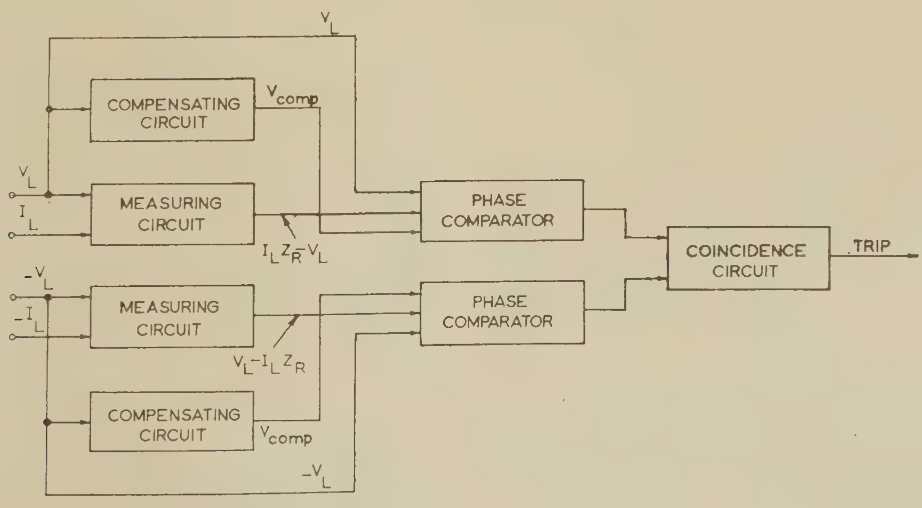


Fig. 5.—Block diagram of relay.

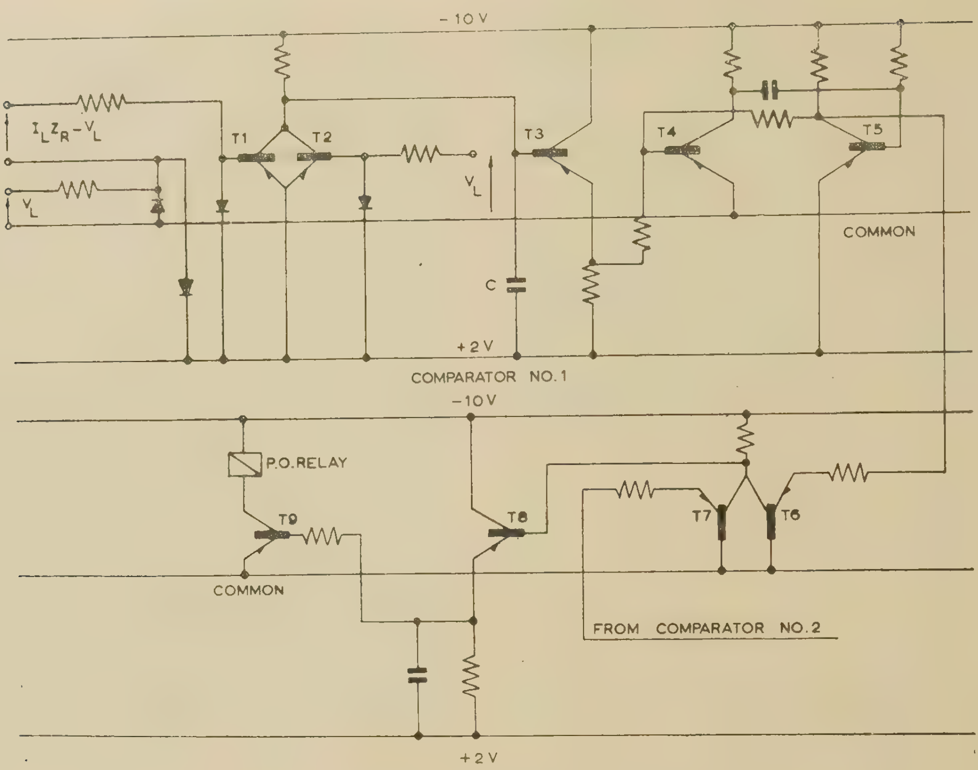


Fig. 6.—Circuit diagram of relay.

(*a*) A curve showing the variation of impedance with fault current, this impedance usually being given as a fraction of the nominal relay setting. This procedure has been discussed by a number of authors, e.g. Hutchinson,[5] and Barnes and MacPherson.[6]

(*b*) A curve showing the variation in operating time of the relay with increase in the distance of the fault from the relay. Since the operating time of a relay is also dependent on the magnitude of the fault current, for a fault at a given position within the protected feeder, a family of such timing curves are necessary for different values of fault current.

Methods such as these result in a large number of seemingly unrelated curves for the purpose of specifying the dynamic characteristics of a relay, and an improved method of presenting dynamic characteristics has been developed by Hamilton and Ellis.[7,8] In this method all the results in (*a*) and (*b*) are presented on one graph known as the accuracy/range chart. This method has been adopted by the authors, and, as a guide to the results which follow, a brief description of the method is necessary.

In Fig. 7, $Z_L$ is the impedance between the relay terminals and the fault, and $Z_S$ is the source impedance between the relay and the generator. $Z_R$ is the nominal setting of the relay, and the ratio $Z_L/Z_R$ expresses the distance of the fault from the relay terminals as a fraction of the total length of the protected feeder. This provides the basis for the definition of accuracy, $x$, such that $x = Z_L/Z_R$.

The ratio, $\dfrac{\text{impedance setting of relay}}{\text{impedance just causing operation of relay}}$, is the value of $x$ at which the relay just operates and defines the threshold accuracy; this should be unity for an ideal relay, but it may be seen from eqn. (2) that unity is unattainable without compensation for the static-bias term $V_b/I_L Z_R$.

Range, $y$, is a property of the composite system formed by the relay setting and the transmission system into which the relay is connected; it is defined as

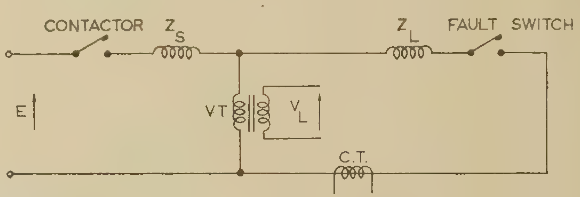


Fig. 7.—One phase of test bench.

$$y = \frac{\text{system source impedance}}{\text{impedance setting of the relay}} = \frac{Z_S}{Z_R}$$

The defined quantities $x$ and $y$ form the basis of the improved method of presenting the dynamic characteristics of impedance relays, as outlined below. The threshold accuracy of the relay is determined, for different values of range, until a point is reached at which the relay no longer operates; with accuracy plotted to a base of range, this curve describes the operating boundary of the relay. The relay operating time is then determined, for many values of $x$ and $y$, within the operating boundary of the relay; points of constant operating time are joined to produce a family of curves. If necessary, a family of constant-current curves may be derived from the formula

$$I_L = \frac{E}{Z_R(x + y)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

A typical chart showing the characteristics of a modern impedance relay is shown in Fig. 8, $y$ being plotted to a logarithmic base for convenience. The constant-current curves are no

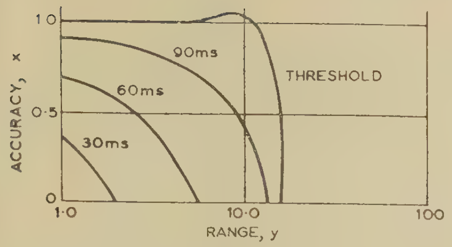


Fig. 8.—Accuracy/range chart for a modern distance relay.

shown on this chart since they are not normally required. It may be seen from the chart that all the data, which were previously demonstrated by means of the large number of curves specified in (*a*) and (*b*) above, are now shown on this one chart. In particular, the operating time at specific points within the protected zone is determined more realistically, under conditions of constant source-impedance, by drawing vertical ordinates on the chart; this is more satisfactory than plotting corresponding curves under conditions of constant current.

**(4.1.2) Tests carried out for Determination of Accuracy/Range Charts.**

The tests described in this Section ascertained the dynamic fault performance of the relay, which was connected as a phase-fault element, i.e. using line-to-line voltages and delta currents. Charts were plotted for the following relay arrangements:

(*a*) Relay connected as a normal phase-fault element.

(*b*) Relay connected as a phase-fault element, but with the polarizing voltage derived from an unfaulted phase in order to preserve the relay characteristics for faults close to its terminals.

(*c*) Relay connected as a normal phase-fault element, but with polarizing voltage derived from a tuned circuit in order to sustain this voltage for faults close to the relay terminals; in this way memory action was provided. In addition, threshold accuracy curves only were determined for this relay configuration under conditions corresponding to the two frequencies 51·4 and 46·4 c/s, thus providing information about the performance of the relay with memory under extreme conditions of frequency variation.

All the tests were performed twice for the cases of zero- and full-transient in the current wave, the two sets of results thus defining the limits between which the relay is required to operate.

## (4.2) Dynamic Polar Curves

Dynamic polar curves were determined from test-bench results by the following procedure. Fault current was obtained from the bench current transformers in the usual way; fault voltage to the relay was obtained artificially from a voltage-dividing network connected to a phase-shifting transformer. This latter network was so arranged that, prior to fault inception, the voltage output was 110 volts; on initiation of the fault, fault current built up in the usual way, whilst the voltage collapsed to a known preset value. Thus simulation of faults in almost any position in the complex plane was possible.

This technique is preferable to the more usual method of secondary injection, which is a static test and gives no information about the behaviour of the relay for the first few cycles after fault inception.

## (5) DISCUSSION OF TEST RESULTS

The accuracy/range charts are shown in Figs. 9–12. Fig. 9 is for the case of the relay normally polarized; Figs. 10 and 11 are for the relay externally polarized from an unfaulted phase, with zero and 100% d.c. transient, respectively, and Fig. 12 is for the relay with memory, and with 100% d.c. transient.

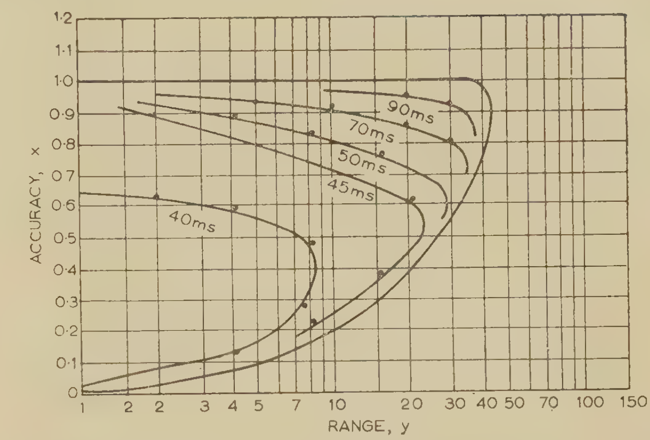


Fig. 9.—Accuracy/range chart for dual-comparator relay, normally polarized.

Average operating times shown.

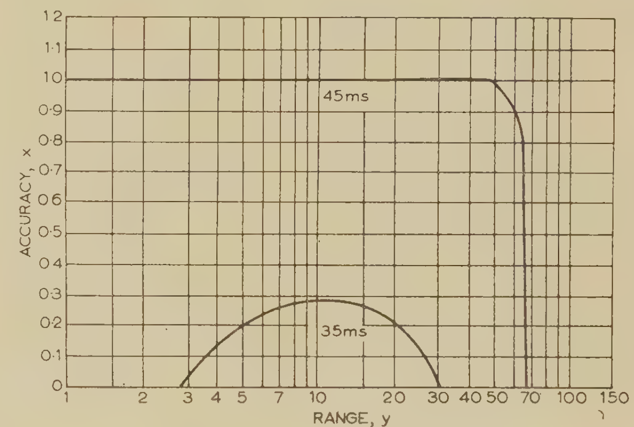


Fig. 10.—Accuracy/range chart for dual-comparator relay, externally polarized.

Operating times for zero d.c. transient shown.

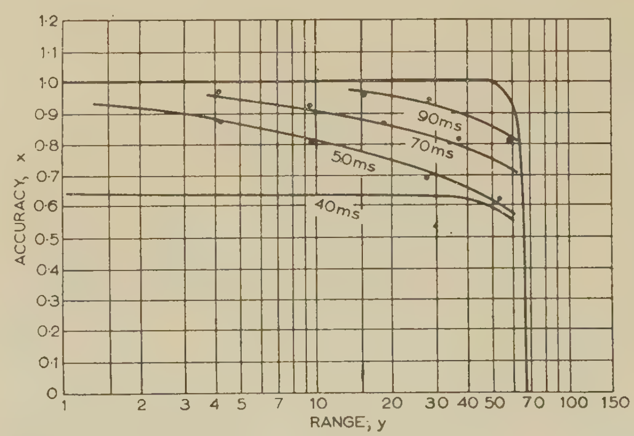


Fig. 11.—Accuracy/range chart for dual-comparator relay, externally polarized.

Operating times for 100% d.c. transient shown.

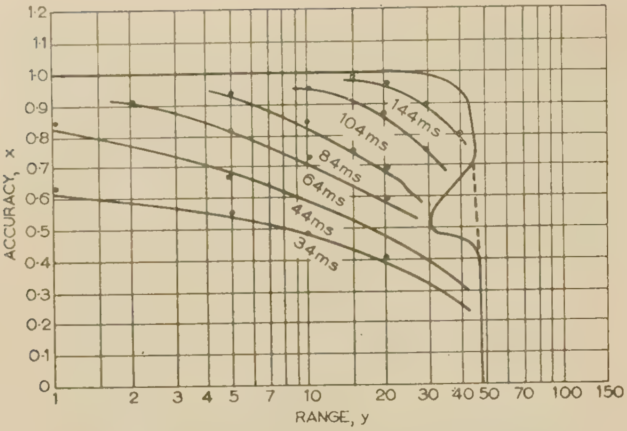


**Fig. 12.**—Accuracy/range chart for relay with memory.
Operating times for 100% d.c. transient shown.

### (5.1) Performance of the Bias-Compensating Network

The effectiveness of the bias-compensating network should first be noted. The threshold accuracy of the relay is constant at unity over a very wide range of source impedance (or fault current), before the relay drops off sharply.

It is instructive to compare the boundary conditions of this relay with that of the original phase-comparator, which has been described in a previous paper.[2] The original comparator had an input sensitivity of 200 mV, which in the opinion of the authors was too small to provide adequate stability. Notwithstanding this high sensitivity, the accuracy of the relay decreased with range and finally fell below the permissible value of 0·9 at a range of 25. The dual-comparator relay described here has the far smaller input sensitivity of 1·7 volts, but has a greater useful range. Furthermore, the relay is far more consistent within this range.

Compensation results in an increase of voltage-transformer burden of 1 VA which is warranted, owing to the improvement in relay characteristics.

### (5.2) Operating Limits of the Relay

With the relay normal (Fig. 9) there is a failure to operate below a certain value of accuracy; the accuracy at which this first occurs increases with range. The effect is expected, and is due to the collapse of polarizing voltage for faults close to the relay terminals; this point has already been dealt with in a previous paper.[2] With the relay externally polarized (Figs. 10 and 11) this effect is eliminated, and the behaviour of the relay is consistent over a very wide range.

With memory applied to the relay, the relay again operates for faults down to the relay terminals (see Fig. 12). There is, however, a reduction in range compared with the externally polarized relay, and also a 'turn-back' effect, which still further reduces the effective range of the relay. This will be considered later when the effect of memory on the accuracy/range chart is discussed. The results for the relay with memory and with zero d.c. transient are not given; they are very similar to those presented in Fig. 10, except that the range is reduced to the value shown in Fig. 12.

### (5.3) Directional Properties of the Relay

In all the cases described above it should be noted that the relay preserved its directional characteristics, no failure being recorded. This is an important point, particularly in the case of the memory tests, in which difficulty is very often experienced from inversion of the mho-circle of the relay.

### (5.4) Relay Operating Time

It might be expected that an instantaneous type of relay would have a sensibly constant operating time. This is not so, and the time delay experienced towards the relay operating boundary is a direct consequence of the elimination of transient overreach due to the duplication of the input circuits. This feature has already been mentioned in Section 2.2 and has been quantitatively analysed in Section 9.1. It may be seen that it is due to the negative transient component in one of the two comparators, and is a function of the system time-constant and the point within the protected zone at which the fault occurs.

### (5.5) Effect of Memory on the Accuracy/Range Curve

The turn-back in the characteristic encountered in the relay with memory may be explained by considering the normally polarized case, Fig. 9. It has already been noted, and may be seen from the curve, that the relay fails to operate below a certain value of accuracy, which increases with range. It thus follows that, for a relay with memory to sustain the polarizing voltage in the event of a near fault, the memory will be required for all faults lying below the non-operate curve in Fig. 9. It may be seen, by superimposing this curve upon the memory chart of Fig. 12, that for large values of range this curve actually lies within the region of time delay. Thus, for a fault falling below the non-operate curve but above the region of no time delay, the relay might fail to operate owing to the fact that the natural system transient has a longer time-constant than the decay time-constant of the memory circuit. The memory time-constant in these tests was deliberately chosen to be as short as possible (approximately 20 millisec). The turn-back effect did not manifest itself in the case of the zero-current transient test, because in this case there was no time delay in relay operation.

### (5.6) Effect of Frequency Variation on the Accuracy/Range Curve

Frequency effects on the accuracy/range curve were estimated by altering the capacitance in the tuned circuit. Complete timing contours were not taken in this test. It was merely verified that the relay operated for all faults in the forward direction and that the relay retained its directional properties by not operating for any faults in the reverse direction. The turn-back effect in the characteristic with full-current transient was again noted. Spot timing points near the operating threshold of the relay once again showed fairly consistent times of operation, and the speed was of the same order as that for the centre frequency. The threshold accuracy/range curves were similar to those of Fig. 10, and were not plotted.

### (5.7) Effect of Frequency Variation on the Dynamic Mho-Curve

The results for the variable-frequency test on the dynamic mho-curves are shown in Fig. 13. Two points arise in connection with these polar curves. First, the relay setting is constant at the nominal line angle. Secondly, a change in frequency introduces a shift in the angular position of the diameter of the mho-circle and, at the same time, the diameter increases. The first point had already been verified by making a variable-frequency test at the nominal line angle at the time when the accuracy/range curves were taken. These showed that the accuracy was independent of the frequency drift over the frequency range chosen. Analysis of the effect of frequency drift is given in Section 9.2. It may be seen that this effect is due to the polarizing voltage being in phase with the restraint voltage only at the resonant frequency. At all other frequencies, the resonant circuit introduces a phase shift which may be positive or negative depending on whether the drift is high or low. Although the diameter increases as the secant of the phase shift in the tuned circuit, and the angular

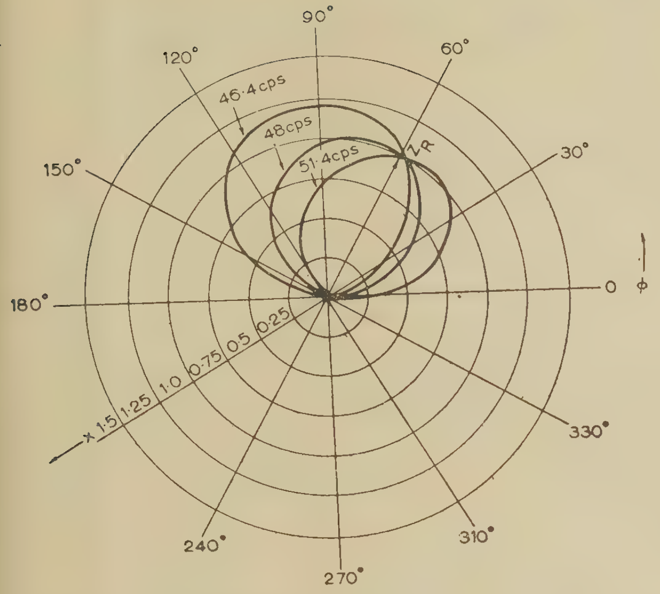


Fig. 13.—Dynamic polar curves for relay with memory.

position of the diameter rotates with the phase shift, it may be seen that the accuracy of the setting angle remains constant. It is important to note that, for a given frequency deviation, both the angular shift of the diameter and the increase of the diameter are proportional to the Q-factor of the tuned circuit. This is a further reason for keeping the Q-factor (and hence the decay time-constant) of the tuned circuit as small as is consistent with reliable operation. A further point to note is that no inversion of the mho-circle takes place; that is, the relay will never operate for faults in the reverse direction, even in the most severe case of frequency shift.

### (5.8) Voltage-Transformer Burdens

The measured burden on the voltage transformers for the relay without memory was 3·5 VA, and for the relay with memory it was 4·5 VA.

The relay setting was variable in 0·65-ohm steps over the range 0·65–26 ohms. At the 26-ohm setting the burden was approximately 3·5 VA and for other values the burden was dependent on the square of the setting.

### (5.9) Relay Operating Time

The operating time of the relay was of the order of three half-cycles in the region of no time-delay. However, a relatively slow Post Office-type relay was used as a slave, since power transistors with sufficient power-handling capacity were not available when the work described in the paper was carried out. With transistors of higher rating the speed of operation will increase, and one half-cycle or more may well be subtracted from the values obtained in the tests described above.

It might be argued that the time delay inherent in this relay is a major drawback, and that superior operating characteristics could be obtained by using two comparators but adopting an alternative criterion such that either comparator should detect a fault in order to produce a tripping signal. In this case the transient appearing at the relay input terminals would have to be attenuated artificially, in order to reduce the transient overreach to an acceptable value. In the opinion of the authors, it is better to make use of a relay with the characteristics described in the paper, since attenuation of the transient appearing at the relay input terminals could still be incorporated; the effect in this case would be to increase the zone of no time-delay. The alternative type of relay would have instantaneous operation over the entire operating range, but would have a certain amount of transient overreach, dictated by the amount of transient filtering incorporated. The dual-comparator relay has instantaneous operation over most of its operating range, and is completely free from transient overreach; circuit configurations require that the relay be exactly one half-cycle slower, in the region of no time-delay, than the alternative form of relay.

## (6) CONCLUSIONS

### (6.1) Transient-Overreach Performance of the Relay

Direct comparison of performance with that of other relays is difficult because of the way in which the tests were carried out. It has been shown[2] that transient overreach in an electronic relay of this type is principally due to the transient appearing at the secondary terminals of the voltage transformers. It is thus necessary to test under exacting conditions, using a high ratio of source reactance to source resistance coupled with a large range, since these factors produce the most severe transient component in the fault-voltage wave.

If a homogeneous line is used for testing, the transient component at the voltage-transformer terminals is reduced to zero, and the input transient to the relay is then only that appearing at the secondary terminals of the current-transforming device, i.e. the transformer reactor. Owing to the inherent filtering of the current transient through the transformer reactor, the maximum value of this transient at the relay input is reduced to about 8%. The consequence of this may be seen by considering the relay described; there would be undelayed operation over about 92% of the relay setting, this being independent of the range at which the relay was working until very near to the relay cut-off point. Within the zone of undelayed operation, the relay operating time would vary between the limits of 20 and 30 millisec, depending on the point in the voltage wave at which the fault occurred. If this relay, under the same test conditions, had the alternative criterion for operation mentioned in Section 6.1, there would be a transient overreach of the order of 8%. It follows that the dual-comparator relay could be set to protect a greater section of zone 1 with correct discrimination than a relay working on the alternative criterion, for which the time of operation would be 10–20 millisec.

The operating times in this discussion are derived on the basis that a Post Office-type relay required 10 millisec to close its contacts from the instant of energizing. However, it should be noted that although 8% transient overreach would not be considered excessive, the argument has been based on the assumption of a homogeneous line; this is not likely to occur in practice, since source reactance is made up of generation equipment, busbars and transformers, all of which tend to have high Q-factors compared with transmission lines.

### (6.2) Further Research

The dual-comparator relay showed a marked improvement over earlier transistor-relay prototypes, particularly with regard to accuracy and the absence of transient overreach. The improvement in performance was, however, accompanied by an increase in the number of transistors required for the relay model. This point dictates further policy with regard to research and development, which should be along the lines of design and rearrangement of individual circuits in order to reduce the number of transistors and, in consequence, the cost of each unit. In particular, the final drive circuits to the slave relay are likely to be improved.

No transistor failures were recorded during the tests, and the

circuits were not critical in the choice of transistors. Circuit behaviour was excellent with the two types of transistor available for test; there is no reason why other types should not perform equally well.


## (7) ACKNOWLEDGMENTS

The authors wish to acknowledge the encouragement given to their work by Professor E. Bradshaw and Mr. I. de Villiers, Chief Engineer (Electrical) of E.S.C.O.M. South Africa, and the co-operation and advice received from the Research and Certification Department of A. Reyrolle and Co., Ltd.


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## (9) APPENDICES

### (9.1) Analysis of Transient Time-Delay

Consider the instantaneous input voltage appearing at one comparator, the d.c. transient being negative. We have

$$v_1 = \hat{V}_L \sin \omega t$$

$$v_2 = \hat{I}_L Z_R \sin(\omega t + \theta - \phi_L) - \hat{V}_L \cos \omega t - K\hat{V}_L \varepsilon^{-t/T}$$

For these tests $\theta = \phi_L$, so that the line voltage and the voltage derived from the line current are in phase for a line-to-line fault. The effect of the transient component in the polarizing voltage will not be analysed.

Thus $$v_2 = (\hat{I}_L Z_R - \hat{V}_L) \sin \omega t - K\hat{V}_L \varepsilon^{-t/T}$$

Fig. 14 shows the waveforms of $v_1$ and $v_2$, the output from the first coincidence circuit and the output from the integrating circuit. It may be seen from this Figure that there is no block output from the coincidence circuit until

$$(\hat{I}_L Z_R - \hat{V}_L) > K\hat{V}_L \varepsilon^{-t/T}$$

Also, the level-detector associated with this comparator can produce no output pulse until the block duration is greater than 90° (electrical). Hence, for an indication,

$$(\sqrt{2})(\hat{I}_L Z_R - \hat{V}_L) > K\hat{V}_L \varepsilon^{-t/T}$$

on the assumption that $K\varepsilon^{-t/T}$ is constant over the interval of block duration.

Let $K\varepsilon^{-t/T} = K'$, then

$$(\sqrt{2})\hat{I}_L Z_R - (\sqrt{2})\hat{V}_L > K'\hat{V}_L$$

$$(\sqrt{2})Z_R - (\sqrt{2})Z_L > K'Z_L$$

$$Z_L[(\sqrt{2}) + K'] < (\sqrt{2})Z_R$$

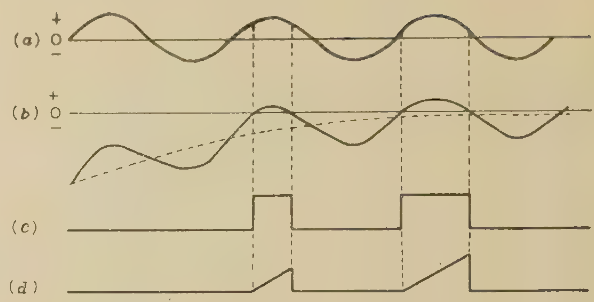


**Fig. 14.**—Waveforms with d.c. transient.
(a) $v_1 = \hat{V}_L \sin \omega t$.
(b) $v_2 = (\hat{I}_L Z_R - \hat{V}_L) \sin \omega t$. Curve of $K\hat{V}_L\varepsilon^{-t/T}$ shown dotted.
(c) Coincidence circuit waveform.
(d) Integrating circuit waveform.

so that $$Z_L < \frac{(\sqrt{2})Z_R}{(\sqrt{2}) + K'}$$

and $$x < \frac{1}{(1 + K'/\sqrt{2})}, \text{ or } K' < (\sqrt{2})(1/x - 1) \quad . \quad . \quad (5)$$

Thus, for a given value of $x$, the relay will fail to operate if the magnitude of $K'$ is greater than the value defined by eqn. (5). In this event, operation would be delayed for one cycle before measurement could once more take place; this process would continue until time had elapsed for $K'$ to decay sufficiently to satisfy the above equation.

A similar analysis shows that the second comparator, in which the d.c. transient is positive, gives an indication for all values of $x$ as defined by the equation

$$x \leqslant 1/(1 - K')$$

In this case, for all values of $K'$ other than zero, the values of $x$ below which the comparator will indicate a fault is greater than unity. This describes the mechanism of transient overreach in an electronic phase comparator. Thus in the dual-comparator arrangement described in the paper, a d.c. transient component will tend to give a false operation in one comparator, whereas in the other comparator it will restrain operation, and thereby cause a delay in overall operating time.

### (9.2) Analysis of the Effect of Frequency Drift

A phase-comparator will have a mho characteristic if the two input voltages are of the form

$$V_1 = k_1 V_L$$
$$V_2 = V_R I_L - k_2 V_L$$

Normally, $k_1$ and $k_2$ are both real numbers representing transformer turns-ratios. In this event, the polar characteristic of the relay is defined by the equation

$$Z \leqslant \frac{Z_R}{k_2} \cos(\theta - \phi)$$

where $Z = V_L/I_L$, and $Z_R = Z_R\angle\theta$

In the case of a relay with a memory circuit for deriving the polarizing voltage, the constant $k_1$ will incorporate the phase shift and attenuation of the tuned circuit, and will therefore no longer be real. Thus $k_1$ may now be written as $k_1 = k_1\angle\alpha$, where $\alpha$ is the tuned-circuit phase-shift.

Thus $$V_1 = k_1 V_L \angle\alpha$$
$$V_2 = Z_R I_L \angle\theta - \phi - k_2 V_L$$

The relay will just operate when these two vectors differ in phase by 90°. For this condition the product of the slopes of the vectors should be $-1$.

Therefore

$$\frac{\sin\alpha}{\cos\alpha}\left[\frac{Z_R I_L \sin(\theta-\phi)}{Z_R I_L \cos(\theta-\phi) - k_2 V_L}\right] = -1$$

Simplifying, $Z = \dfrac{Z_R \sec\alpha}{k_2}\cos(\theta - \phi - \alpha)$

It is clear from this expression that, at the nominal line angle, $Z = Z_R/k_2$ and is constant for all values of $\alpha$. Thus, at the nominal line angle, the relay setting is a common chord to the family of polar circles, and for the case of $\alpha = 0$ only it is a diameter and reverts to the familiar mho-circle.

The phase shift $\alpha$ between the polarizing voltage and the line voltage is introduced by the resonant tuned circuit. Referring to the memory circuit shown in Fig. 15.

$V_L$ = Fault voltage

$V_P$ = Polarizing voltage.

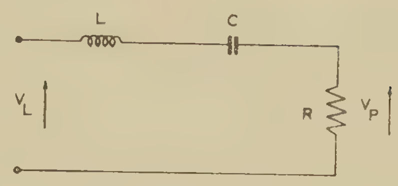


Fig. 15.—Memory circuit.

Then $V_P = \dfrac{V_L}{Z}R$

If $Z = Z\angle\psi$, then $V_P = V_L\dfrac{R}{Z}\angle -\psi$

In the expression above,

$$\alpha = -\psi$$
$$\tan\psi = \omega L/R - 1/\omega CR$$
$$= Q(\omega/\omega_0) - Q(\omega_0/\omega)$$
$$\tan\alpha = Q(\omega_0/\omega - \omega/\omega_0)$$

Thus $\alpha$ is a function of the Q-factor of the tuned circuit, and of the frequency drift.

# DISCUSSION ON 'FLAT PRESSURE CABLE'*

## SOUTH MIDLAND SUPPLY AND UTILIZATION GROUP, AT BIRMINGHAM, 13TH FEBRUARY, 1956

**Mr. H. M. Fricke:** I would like more information about the coefficient of expansion of the compound used for impregnation. The use of conducting paper on the strand is a very logical step towards producing radial stress only. The long life of paper cables is an encouragement to supply authorities to expect 50 years' use. The use of copper-bearing alloys external to the lead sheath imposes an important duty on the anti-corrosion protection to avoid electrolytic action.

**Mr. P. M. Martin:** I imagine that the pressure inside the cable depends on the restraint applied to the lead sheath, which, in turn, depends on the tension applied to the copper wire during winding. However, there does not seem to be any precise means of applying a definite tension to this wire. I would like to have the authors' comments on this point.

In the jointing at sea of two lengths of cable, the jointed cable was slid over a ramp. I do not know what happens to the piece of U-shaped cable when it gets to the bottom of the sea, but I should imagine that it is rather liable to kink. Is that, in fact, a serious source of trouble?

**Mr. H. C. Fox:** In Section 8.1 it is stated that 'The bending radii are the same as for single-core cables for bends about the major axis of the cable'. What relative sizes of single-core cables are referred to?

**Mr. J. P. Cranmer:** It is well known that 3-phase conductors arranged in a straight line have unequal inductances. Is it necessary to transpose the cable cores at joint boxes (see Fig. 4)?

What provides the buoyancy for the cable drum?

**Mr. G. S. Buckingham:** The cable is a member of the family of oil-filled cables. The normal orthodox oil-filled cable has to have pressure tanks along its length for accommodating the expansion and contraction of the oil under load conditions. These are a nuisance, and it is obviously an advantage to be able to do without them. The present cable accommodates the expansion and contraction of the oil by the movement of the lead membrane, comprising the flat side of the cable. The authors have a wealth of experience and knowledge of the movement of lead as a membrane. It is not a metal that we usually regard as safe under constant flexing, and I would like the authors to amplify the information they have given us. It is not satisfactory if the flexing is going to bring the useful life of the cable to an end after 40 years.

With the oil-filled-cable installations in the Midlands we have occasional trouble at terminations which are installed on high lattice towers, because of the excessive variation in air tem-

* MØLLERHØJ, J. S., MORGAN, A. M., and SUTTON, C. T. W.: Paper No. 1884 S, July, 1955 (see 103 A, p. 134).

perature resulting from extremes of weather in the winter and summer. These very wide ranges of temperature are quite beyond those experienced with cable buried in the ground, with the result that oil-pressure indicating devices operate incorrectly. I would expect the same trouble with the copper-tube terminations described, and I would like to know whether alarms are installed on these cables to give any indication of whether the cable is operating at its correct pressure.

**Mr. R. A. York:** If the movement of compound in the cable is mainly radial, are the alarms of any great use, and will they be of any assistance if there is damage to the cable and a leak in it? Would one expect to get an alarm before the cable broke down; or if there was a hole in it, would it be possible for electrical breakdown to occur before there was an alarm indicating that the pressure was low?

Fig. 8 indicates that there is very little difference between the temperature of the outer and inner conductor. Is there any appreciable difference in the rating of the cable when it is laid with its major axis horizontal and vertical?

Can the authors explain the function of the stop joints on the cable?

I am not yet entirely convinced that the cable is better than any other for 33 kV or 66 kV working; it is satisfactory, but has it any economic advantage? Have the authors any figures of the cost, say in kVA per mile, as against other cables? Is it more economical for short lengths because it does not need such complicated terminations?

**Mr. A. Shaw:** Since the flat sides of the cable are used as a diaphragm (see Section 1.1), is there not a danger of the sides being permanently distended at the lower points of the route, thus allowing loss of oil at the higher points?

I can see a great deal of difficulty in passing the copper tubes over the single cores at terminating positions (see Section 5.2.2). The cores will have to be laid out straight for this operation. Often where the cables terminate in a transformer bay there is very little room for this. In the case of 132 kV cables the sealing ends may be as far apart as 9 ft. It will be quite a problem to set the copper tubes between the splitter box and the sealing ends. Is any precaution taken against compound migration from the sealing ends?

In Section 4 it is stated that long lengths of the cable can be manufactured. Could the authors give a maximum length for, say, a 132 kV 3-core $0\cdot4\,\text{in}^2$ cable?

A pressure of $5\,\text{lb/in}^2$ is maintained on the cable. What method is used to maintain this pressure?

In Section 6.2.1 it is not stated how much greater the losses are over the conventional 3-core cable when a single length of cable is used in a system and no transposition of the cores is possible.

During jointing operations the cable must be frozen some distance from the joint to enable plumbing to be carried out (see Section 8.1). Are the authors satisfied that the short length of cable on each side of the joint is fully impregnated at the same time as the joint is filled?

We are not told how the oil pressure in the cable is adjusted after several load cycles. In long cable systems which might pass over high points as well as low, is the cable sectioned off by the use of barrier joints? If not, what are the maximum and minimum pressures permissible on a system?

**Messrs. J. S. Møllerhøj, A. M. Morgan and C. T. W. Sutton** (*in reply*): With reference to the comments of Messrs. Fricke, Martin, Fox and Cranmer, we would state that the coefficient of expansion of the oil used is similar to that used in oil-filled cables. The tension applied to the copper wires used for retaining the corrugated membraneous strip is not critical. The initial hydraulic pressure takes up any small amount of slack in the retaining wire, after which the pressure can be adjusted to the conditions of service.

No damage due to kinking has been experienced when the cable is slid over the ramp of the ship. The joint was supported upon a rigid steel frame, and, in the case of the installation in 1951, the joints were inspected by a driver after laying.

The bending radii are the same as those for a single-core cable of diameter equal to the dimension of the flat cable across the minor axis.

Transposition of cores is rarely necessary, and when required the adjustment can be made in the joint.

The floating drum is so dimensioned that, when it carries a full load of cable, it will float in a stable position. As the cable is laid the drum tends to rise in the water, and to counteract this water is taken in to the drum to provide ballast.

In reply to Mr. Buckingham, installations in Scandinavia are subject to extremes of weather which are greater than those experienced in this country, and no weaknesses have been revealed.

Alarms are installed at each end of an installation and consist of a pressure-sensitive switch, which operates at the maximum and minimum pressures permissible for safe operation of the cable.

When a leak occurs in the cable, there is longitudinal movement of oil, and under these circumstances the alarms, which are located generally at the terminations or trifurcating boxes, will operate effectively. When on full load, the cable acts as an oil reservoir, and should a leak occur, there is an adequate oil reserve to prevent low pressures occurring in the middle of the cable before an alarm is given at the termination. For example, when a cable was cut for examination, oil continued to be forced out of the cable two days after cutting. Generally, the cable is laid with its major axis horizontal, so that full advantage can be taken of the preferential heat-dissipating properties of the cable in this position.

There is, however, a small reduction in the rating of the cable when laid with its major axis in the vertical position, but this occurs only when rounding bends.

The stop joints are installed only in exceptional cases, i.e. to limit hydrostatic heads on inclines. There is no need to sectionalize as in long horizontal installations of conventional oil-filled cable.

Many installations have been made with the cable terminations in transformer bays. Copper tubes have been used effectively without damage to the installation, and undue complications in erection, even when the available space is limited. No oil migration from the sealing ends can occur, as they are effectively sealed from the atmosphere and are at the same pressure as the cable.

The short lengths of cable on each side of the joint between the freezing boxes are fully impregnated when the joint is filled. Vacuum filling is used, and a vacuum-drop test is made on the joint to ensure that all gas has been evacuated from the joint before the filling operation is commenced.

Oil-pressure adjustments are carried out by the introduction of de-gasified oil into the cable. At present, a maximum permissible pressure on the system is $90\,\text{lb/in}^2$, but with modification in design, cables have been made which will operate up to $150\,\text{lb/in}^2$.

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Paper No. 2145 M
Oct. 1956

# A NOVEL HIGH-VOLTAGE PEAK VOLTMETER

By W. P. BAKER, B.Sc.(Eng.), Associate Member.

(*The paper was first received 7th March, and in revised form 17th May,* 1956.)


## SUMMARY

The paper describes a new type of high-voltage peak voltmeter which is free from the errors normally encountered in such instruments. The instrument, which includes rectifiers in a feedback loop, is sensibly independent of rectifier characteristics, and is particularly suited to the measurement of high voltage in the presence of corona discharge.


## (1) INTRODUCTION

Of the measures of an alternating voltage (peak, r.m.s., average, etc.) the double-amplitude-peak (d.a.p.) value seems to be the most important in the testing of insulation over the range of waveforms normally encountered, including those associated with conditions of corona discharge. This is demonstrated in Table 1, which shows the results of many electric-strength tests on a few insulating materials, both with applied voltages of nearly sinusoidal waveform (peak factor 1·36) and of peaky waveforms (peak factor 3–5·5). Absolute voltmeters, however, measure the r.m.s. value, and for this reason the Chubb–Fortescue peak voltmeter has been extensively used in the past, despite two very serious drawbacks. These are the frequency dependence of the indication and the errors introduced by heavy discharge on the high-voltage system (which may frequently be of the order of 5% and have been known to be as great as 20%); a less important shortcoming is the error introduced by multiple peaks in the voltage waveform.

Experience with the Chubb–Fortescue instrument at voltages of 100 kV or more has indicated the need for a better form of peak voltmeter which is free from the above disadvantages.

Table 1

Results of Breakdown Tests with Sinusoidal and Peaky Voltage Waves

| Material | Thickness range | Total number of breakdowns | Electric strength | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Peaky waveform | | | | Nearly sinusoidal waveform | | | |
| | | | Rapid | | Step-by-step | | Rapid | | Step-by-step | |
| | | | r.m.s. | crest | r.m.s. | crest | r.m.s. | crest | r.m.s. | crest |
| | mils | | volts/mil | volts/mil | volts/mil | volts/mil | volts/mil | volts/mil | volts/mil | volts/mil |
| Varnish films, Type 1 | 1·5–1·9 | 129 | 510 | 2 070 | 430 | 1 670 | 1 500 | 2 040 | 1 190 | 1 640 |
| Varnish films, Type 2 | 1·6–2·1 | 49 | 620 | 1 930 | 525 | 1 860 | 1 500 | 2 040 | 1 230 | 1 680 |
| Varnish films, Type 3 | 2·1–2·5 | 30 | 650 | 1 910 | 460 | 1 680 | 1 560 | 2 140 | 1 220 | 1 670 |
| Black varnished tubing, 600 V Grade | 8·5–9·0 | 18 | — | — | 30 | 160 | — | — | 120 | 160 |
| Black varnished tubing, 1 000 V Grade | 18 | 26 | 30 | 140 | 30 | 120 | 100 | 140 | 70 | 100 |
| Double-thickness Kraft paper | 9·8–10·7 | 40 | 80 | 300 | 70 | 290 | 230 | 310 | 220 | 290 |
| Triple-thickness greaseproof paper | 7·2–7·5 | 40 | 110 | 400 | 110 | 390 | 280 | 390 | 280 | 380 |

Rapidly applied tests: $\dfrac{\text{mean of peaky wave peak values}}{\text{mean of sine wave peak values}} = \dfrac{1\,125}{1\,170} = 0{\cdot}96.$

Step-by-step tests: $\dfrac{\text{mean of peaky wave peak values}}{\text{mean of since wave peak values}} = \dfrac{880}{850} = 1{\cdot}03.$

The ratios of the r.m.s. values will obviously be far from unity.

## (2) METHODS OF MEASURING PEAK VOLTAGES

### (2.1) The Chubb–Fortescue Voltmeter

The method shown in Fig. 1 involves the measurement of the average charging current of a high-voltage series condenser, C. Current will pass through the instrument, M, during the interval between a voltage maximum and the following voltage maximum of opposite polarity.

If the charges on the high-voltage condenser at these two instants be $-V_pC$ and $+V_pC$, the total change of charge, $\Delta Q$, is $2VC$ and the average current passing through the instrument during this time is

$$\frac{\Delta Q}{\Delta t} = \frac{2V_pC}{1/2f}$$

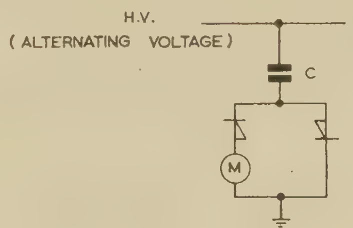


**Fig. 1.**—Chubb–Fortesque high-voltage peak voltmeter.

The instrument reading would be one-half this value, because the current is passed only on alternate half-cycles, so that

$$I_{av} = 2V_pfC$$

and the peak voltage $$V_p = \frac{I_{av}}{2fC}$$

Written contributions on papers published without being read at meetings are invited for consideration with a view to publication.

Mr. Baker is with the Metropolitan-Vickers Electrical Co., Ltd.

If sine waveforms of voltage are assumed and the instrument is calibrated to read r.m.s. values, then

$$V_{r.m.s.} = \frac{I_{av}}{2\sqrt{2}fC} = \frac{2\cdot 22 I_{av}}{\omega C}$$

since $$\pi/\sqrt{2} = 2\cdot 22$$

### (2.2) Other Methods

Apart from the Chubb–Fortescue voltmeter and a sphere-gap, all the generally known methods of measuring peak voltages involve charging a condenser to a direct voltage approaching the peak value by rectifying the alternating voltage. The differences between these methods become, then, differences in measuring the direct voltage, and the two better known forms are shown in Fig. 2. Each form has the same errors.


(*a*) (*b*)

Fig. 2.—Two forms of diode peak voltmeter.

First, the voltage across the capacitance falls by $\delta V \simeq It/C$ between successive peaks, where

$I$ = Average current flowing in the d.c. circuit.
$t$ = Time between successive charging peaks.
$C$ = Reservoir capacitance.

This error results in the average voltage being lower than the peak value by approximately $It/2C$. Moreover, the current from the diode has a positive value when the anode voltage is slightly negative so that there is a zero error. This is a function of the cathode temperature and cannot readily be corrected; however, if the voltage is more than a few hundred volts, the effect may be negligible.

## (3) APPLICATION TO HIGH VOLTAGES

The highest voltage to which either of the two foregoing methods may be applied is limited by the peak inverse voltage (p.i.v.) of the diode. For small valves this limit corresponds to about 7 kV r.m.s.; above this voltage the cost of the commercially available diodes rises rapidly.

The voltage range may be extended by the use of a voltage divider, but the input impedance of the diode may upset the divider ratio unless the divider has a low resistance. A resistance divider, in turn, introduces difficulties, because the stray capacitances are not easy to take into account. On the other hand, a capacitance divider has an accurately measurable ratio at high voltages, but the lack of a d.c. path for the diode feed may upset this measured ratio.

A possible solution to this problem is to place between the divider output and the rectifier an amplifier wherein, by suitable feedback arrangements, the gain may be stabilized at 1·00 to within 0·1%, and the input impedance may be made high and the output impedance low. Unfortunately, this still leaves unsolved the problem of the zero-voltage anode current of the rectifiers, since the amplifier may deliver only about 100 volts peak in class-A operation.

The solution offered here is to include the rectifiers in a feedback loop, so that the amplifier is actuated by the difference between the indicated voltage and the input voltage.

## (4) THEORY OF THE NEW METHOD

An outline of the new method is shown in Fig. 3, and the general operation is as follows.

A small signal is received by the input grid $G_1$ of the amplifier, and, after voltage and power amplification, the signal is converted

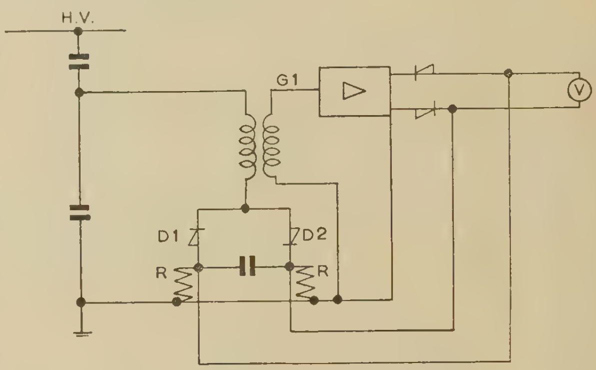


Fig. 3.—New peak voltmeter.

to a balanced direct voltage and applied to the biasing condenser C of the input diodes ($D_1$, $D_2$). The signal voltage developed across the transformer primary is therefore the difference between the d.a.p. voltage received from the potential divider and the direct biasing voltage on C.

If the ratio $A = \left[\frac{\text{Output voltage (d.c.)}}{\text{Peak-to-peak input at } G_1}\right]$ defines the gain of the amplifier, then the d.c. voltmeter should indicate $\left(1 - \frac{1}{A}\right) \times V_{a.c.}$ (d.a.p.) and an error of less than 0·1% should be attainable.

## (5) THE PRACTICAL CIRCUIT

Certain refinements to the circuit shown in Fig. 3 are necessary, and these will be considered in turn.

### (5.1) D.C. Circuit

The direct voltage produced by the rectifier circuit would require smoothing, and in order to minimize the danger of instability and to ensure equal loading of the input diodes under conditions of circuit or input asymmetry, a ripple waveform symmetry, as shown in Fig. 4, is necessary. A ripple voltage

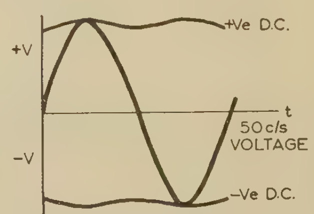


Fig. 4.—Comparison of direct sinusoidal waveforms.

below 0·001% is readily obtainable, and a value of 0·01% should be tolerable. The temperature of the diode cathodes may be reduced to a low value to reduce the zero-voltage anode current to a negligible value.

### (5.2) Power Output Stage

In order to produce the direct-voltage waveform shown in Fig. 4, a push-pull output stage is necessary. The input impedance of a diode is highest when it is fed from a tuned circuit and the value is then approximately $0\cdot 5R$, where $R$ is the resi

tance on the d.c. side. The best conditions of impedance match are obtained with a voltage-doubler circuit, so that the output stage would consist of two pentodes feeding a tuned transformer, the secondary of which would feed into a Greinacher voltage-doubling rectifier circuit. (This circuit satisfies the d.c. waveform requirements in Section 5.1.)

### (5.3) The Input Circuit

The stray capacitance from the rectifier wiring and from the primary winding to earth would enable a current to flow in the

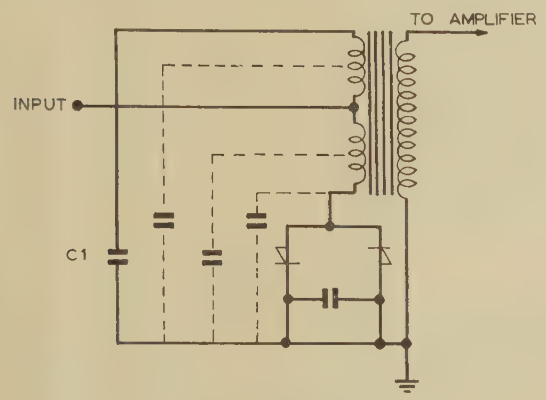


**Fig. 5.**—Balanced input circuit.

transformer primary several times that required to actuate the amplifier and d.c. circuit, so that a balanced input, as shown in Fig. 5, is necessary. The stray capacitances are also shown in this Figure. If $C_1$ is made variable in capacitance and phase angle, complete balancing of the input is possible, but only at one frequency. The critical nature of the balancing condition is a serious disadvantage, and an alternative arrangement is preferred. Because under normal working conditions the diodes conduct only on the peaks of the applied voltage wave, the voltage appearing at the input grid is very rich in third harmonic. If the input circuit is approximately balanced for capacitance, the voltage at the input grid due to unbalance in phase angle will be mainly of fundamental frequency, since the phase angle balance improves with increase in frequency. The amplifier may be made to discriminate against the signal resulting from unbalance by being tuned to 150 c/s, and in this way complete stability is easily attained. This artifice also renders the instrument insensitive to spurious signals arising from heavy discharge.

### (5.4) The Amplifier

A complete circuit of the equipment is shown in Fig. 6.

An EF.37A low-noise pentode has a nominal gain of 152, with a line voltage of 200 volts, when the anode and screen loads are 220 and 680 kilohms respectively. By direct coupling of this stage to a phase-splitter, having equal anode and cathode loads and driving a fraction of the anode load of the EF.37A from the phase-splitter cathode, the dynamic impedance of the EF.37A anode load may be increased nearly tenfold, resulting in an increase in voltage gain of about five, i.e. to about 750 (a more complicated circuit giving a nominally greater improvement has been described by Briggs and Garner*). It should be noted that the cathode load of the phase-splitter consists of 15 kilohms in the cathode d.c. circuit with the 33-kilohm resistance of the first-stage anode circuit in parallel making a dynamic load of 10 kilohms, as in the phase-splitter anode circuit.

By correct matching of the output, a d.a.p. a.c./d.c. gain o about 50 is available from the output stage without resorting to class-B conditions.

The input transformer has a turns ratio of 1 : 12, so that the nominal total gain is 12 × 750 × 50 d.a.p. to d.c., i.e. 150 000 r.m.s. to d.c. Because the amplifier is tuned to the third harmonic, the power gain is reduced to 1/3, and the voltage gain becomes about 90 000, which is more than adequate.

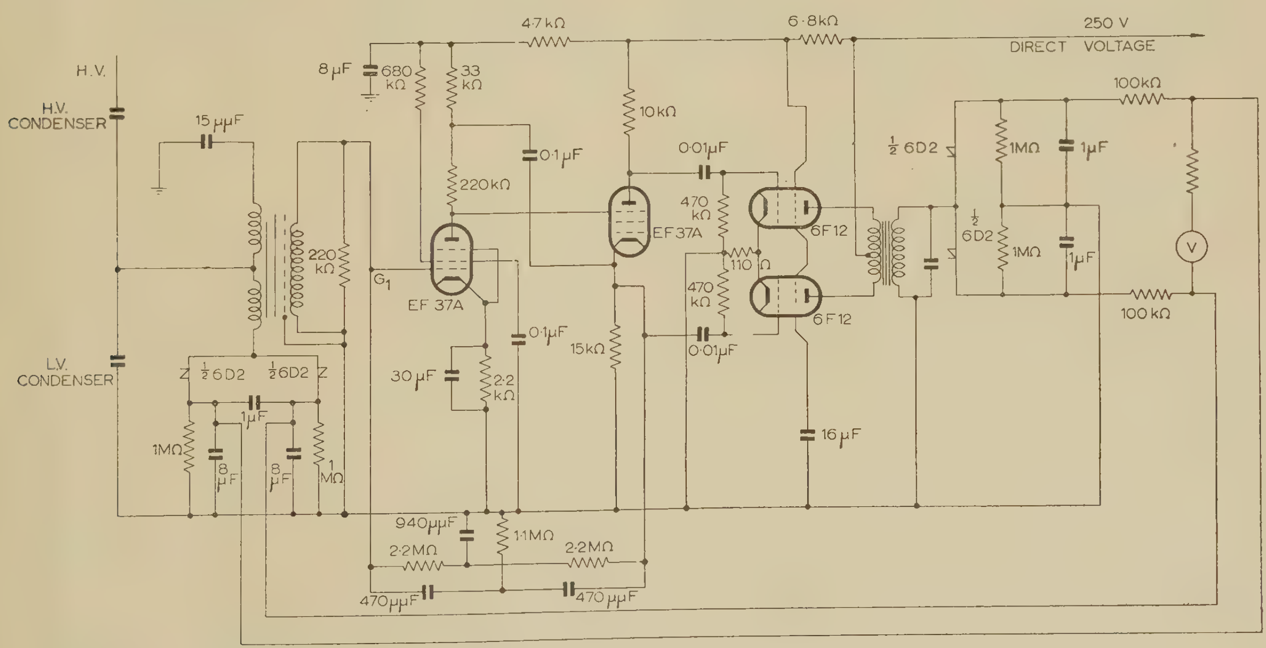


**Fig. 6.**—Complete circuit of peak voltmeter.

A parallel-T feedback network across the EF.37A is used to

* Briggs, G. A., and Garner, H. H.: 'Amplifiers' (Wharfedale Wireless Works, Bradford, 1952).

tune the amplifier. Attenuation in the feedback path reduces the nominal Q-factor by about four times to

$$\frac{1}{4}\left(\frac{A_1 + 1}{4}\right) = 45$$

where $A_1$ is the gain of the EF.37A stage (= 750).

## (6) PERFORMANCE

Because the new instrument represents an attempt to achieve a higher order of accuracy than is normally available for high-voltage measurement, the calibration had to be calculated from measurements on isolated parts of the instrument. First, a voltage divider having ratios of 1 000, 2 000 and 4 000 : 1 was formed by adding switched capacitances to the low-voltage end of a standard compressed-air condenser of $16 \cdot 5\pi$ $\mu\mu$F rated at 250 kV r.m.s., and the output from the voltage divider was fed into the new peak voltmeter. A d.c. microammeter, scaled from 0–50 microamperes, was fitted with a series resistor to convert it into a voltmeter of 141·4 volts full-scale deflection. This voltmeter was fed from the d.c. output of the new voltmeter.

It was first necessary to establish that the complete instrument was free from gross errors, and this was done by means of a sphere-gap test.

The capacitance of an oil-impregnated condenser was measured over a range of voltage, and a rectifying milliammeter was calibrated on a 50 c/s supply. With these two components, a Chubb–Fortescue peak voltmeter was set up to measure up to 50 kV r.m.s. A comparison of the two instruments was then made up to 40 kV. The frequency of the supply was measured, and the waveform was observed to be free from spurious peaks by means of a cathode-ray oscillograph. This comparison is shown in Table 2, which demonstrates that the new instrument is free from serious errors.

The zero error of the instrument due to zero-anode-voltage diode current was 0·2%. The effect of including the diodes in the feedback loop was evaluated by breaking the amplifier circuit, removing the input EF.37A valve, and observing the error introduced by having the voltage divider feed the instrument directly through the input diodes. This test was made at 40 kV, and the indication fell by 45%.

The instrument has been in use for general-purpose work with a 200 kV r.m.s. testing set since the beginning of 1955, and has proved reliable during that time.

**Table 2**

COMPARISON OF NEW AND CHUBB–FORTESCUE PEAK VOLTMETERS

| Nominal voltage | Chubb–Fortescue voltmeter | New voltmeter |
|---|---|---|
| kV | kV | kV |
| 10 | 10·55 | 10·6 |
| 20 | 20·9 | 20·8 |
| 30 | 31·5 | 31·5 |
| 40 | 41·95 | 42·0 |

## (7) CONCLUSIONS

A new type of voltmeter has been developed which is free from many of the shortcomings of well-established methods of high-voltage measurement. By the inclusion of the rectifiers in a feedback loop, the performance of the instrument is very nearly independent of the characteristics of the rectifiers. The low-voltage part of the instrument may have applications in precise voltage measurements up to 50 volts, but for such purposes it is rather restricted, owing to the need for tuning.

## (8) ACKNOWLEDGMENTS

The author wishes to thank Dr. Willis Jackson, Director of Research and Education, and Mr. B. G. Churcher, Manager of the Research Department, Metropolitan-Vickers, Electrical Co., Ltd., for permission to publish the paper.

# DISCUSSION ON 'THE SUPPLY OF ELECTRICITY IN THE LONDON AREA'*

*Before the* NORTH-WESTERN SUPPLY GROUP *at* MANCHESTER 18*th October*, *the* NORTH-EASTERN CENTRE *at* NEWCASTLE UPON TYNE 28*th November*, 1955, *the* WESTERN SUPPLY GROUP *at* CARDIFF 16*th January*, *the* SOUTHERN CENTRE *at* HOVE 25*th January*, *and the* SOUTH-EAST SCOTLAND SUB-CENTRE *at* EDINBURGH 21*st February*, 1956.

**Mr. H. Shackleton** (*at Manchester*): Belted cables were also being laid in Manchester in 1923 for operation at 33 kV, but as soon as these began to carry any appreciable load, faults began to occur. By 1929, 132 faults had been experienced. Some undertakings, like those in London and the Lancashire Electric Power Co., de-rated such cables on voltage, but in Manchester it was decided to continue operation at 33 kV but to restrict the current carried by the cables. For example, $0 \cdot 3\,in^2$ 3-core cables were limited to 170 amp in December and 156 amp in September. The Lancashire Electric Power Co. went even further, in that they tried to arrange that the current on certain cables should never fall below a certain value. These restrictions of the demands made on the early belted cables served to reduce fault incidence very considerably. By 1927 screened 33 kV cables were being laid in Manchester, and the number of faults on these cables was infinitesimally small. Occasionally a joint fault was experienced. It is to be noted that, when Hochstadter presented a paper in Manchester in the early 1920's and suggested the screened cable, his proposal was not very well received. In view of the unsatisfactory performance of belted cables, however, the supply engineers were probably prepared to try anything which would reduce fault incidence, and so H-type cables were installed and found very satisfactory.

In those days fault-clearance times were appreciably longer than they are at present, and protective gear was not as reliable as the modern equipment. It was not uncommon, therefore, for two or three circuits to open in addition to the one on which a fault had developed.

Earth-fault alarms, operated from current in the neutral-earth connection at the generating station supplying the 33 kV system, proved invaluable in those days in confirming whether or not earth current had flowed when a circuit-breaker had opened somewhere on the system. One could not always be certain whether the switch opening was due to an actual fault, to protective-gear instability or to a transient fault, owing to a momentary breakdown in a joint which afterwards sealed itself. Quite a number of transient faults of this type were experienced. In the Manchester network it was necessary to install air-cored reactors in a number of the old belted feeder circuits which were required to work in parallel with the newer H-type cables of much higher rating.

As in London, there were extensive installations in Manchester of bare copper strip installed on insulators in brick culverts and vulcanized bitumen (v.b.) cables in wooden troughing. For many years these systems worked as 5-wire d.c. networks, having a voltage of 200 volts between adjacent conductors and 400 volts between alternate conductors. Flexibility of the network was enhanced by the use of Wordingham pillars installed in manholes at strategic points in the city. The pillar comprised an insulated stand on which were mounted three to seven brass rings, which could be split, to which were connected the appropriate cores of the various distributors. In the late 1920's and early 1930's the v.b. cables were responsible for many street fires and explosions, and in a few instances, bitumen gas entered houses on the route of the mains supply and caused some deaths. Subsequently most of the v.b. cables were replaced by paper-lead ones, but quite a lot of the v.b. system was changed over to a.c. working, and this introduced a greater danger than had existed with d.c. working. It had been possible, with a 3-wire d.c. system, to ascertain the fault resistance of the network at any time, thus enabling potential weaknesses on the system to be located during the next night shift. No such method was available for testing the fault resistance of an a.c. 4-wire system whilst live, and so serious faults developed. The 1934 Electricity Supply Regulations actually called for the frequent and regular testing for insulation resistance on 'lines which are not completely enclosed in a continuous metal sheath efficiently connected with earth', although there was no known method of testing without isolation from the system. A method was, however, devised in Manchester† which enabled faults to be detected in many cases before they became serious. It may be of interest to note that a small amount of copper strip is still in service in the city area, and that some lengths of v.b. cable are still in use in suburban districts not adjacent to houses. In addition, over 400 Wordingham pillars are still in service in the city.

It is rather surprising to note that there are quiet spots in London where even transformer noise can be heard.

Bearing in mind the short distances for transmission and distribution in the London area, a 20% range of voltage regulation seems somewhat high. It seems to confirm the view I have held for a long time that the greater part of the range of voltage-variation equipment is required to compensate for the undesirable voltage characteristics at the generating stations and not for voltage drop on the Board's system. Voltage at the generating-station busbars is often high when we would wish it to be low, and it is a minimum when we would like it a maximum, i.e. at time of heavy load. I feel that much more could be done by the C.E.A. to improve voltage at Grid intake points.

I agree with the author that p.i.l.c. cables properly installed are one of the most reliable items of electrical equipment, but it is asking rather a lot of certain 30-year old $6 \cdot 6$ kV cables if we up-rate them to 11 kV. Experience in the north-west has been far from happy. As a rule, performance for a few months after up-rating is good, but then faults begin to occur, the quiescent period being shorter, as one might expect, for cables of small sectional area. It has been our practice, in view of the large number of faults occurring, to insert a drum length of new cable at each fault position, so that the old cable is gradually being replaced and the fault incidence begins to diminish. At the worst period, however, the rate of fault incidence has been of the order of 140 faults per 100 miles of cable per annum. 80% of the faults involved failure of cable insulation, whilst 20% occurred in joints and/or terminations.

**Mr. J. Gillions** (*at Manchester*): The transformer units and associated switchgear used on the radial interleaved system of distribution appear to have an economic advantage in that they avoid the high costs associated with conventional distribution

* IRVING, D. B.: Paper No. 1748 S, November, 1954 (see 102 A, p. 808).

† CARR, J. L., and SHACKLETON, H.: 'Network Fault Resistance', *Journal I.E.E.*, 1935, 76, p. 222.

substation equipment, but I would suggest that the scheme does not have much to commend it from the point of view of operation and maintenance. No sectionalizing facilities are provided on the h.v. distributor, and therefore a large reserve of plant is necessary on adjacent distributors to prevent prolonged interruptions of supply in the event of a fault developing on the manually-operated h.v. unit. Can the author state whether the earthing arrangements provided on the h.v. units are suitably rated so that they may be closed with safety on to an inadvertently live feeder?

The fault analysis given in Section 6.4, although rather brief, does indicate in some measure the operational performance of a large city distribution system. It is interesting to compare the author's statistics with comparable ones of the North Western Electricity Board. Since vesting day, some 22% of all underground faults and 40% of overhead faults were due to agencies over which the Board has little or no control. The fault incidence on the 33 kV network is two per 100 circuit miles per annum on the underground cable system compared with 20 faults per 100 circuit miles per annum on the overhead system. The 33 kV cable faults were divided equally between those caused by cable insulation failure in joints and terminations.

The assessment of faults on the 11 and 6·6 kV cable systems compares with the author's figures in that 0·4 fault per 100 circuit miles per annum was due to failures in joints and terminations and a similar rate due to mechanical damage. The mechanical damage is divided equally between that caused by driven objects and by failure of lead sheaths due to the movement of cables in ducts. It will be interesting to have the author's experience of this latter fault.

I should have welcomed similar statistics relating to substation performance, particularly with regard to the interruptions of supply due to the over-current and mal-operation of protective-gear classifications. It would also be interesting to know the fault-clearance efficiency of the circuit-breakers and protection gear employed on the various systems.

The method of assessing the electrical efficiency of the distribution system by comparing the number of kilowatt-hours sold to consumers and the number purchased can only be a yardstick, but it is interesting to note that, on this basis, the efficiency of the North-Western Electricity Board distribution system has increased from 91·1% in 1948 to 91·6% in 1955.

**Mr. L. Wain** (*at Manchester*): In Section 3.1 the author mentions the consideration given to the long-term plans of the Board. It would be of interest if he would indicate how far ahead these plans look. My experience is that the period over which the plan is considered can be very critical in regard to the voltage at which reinforcement is carried out. To quote a recent example: Looking ten years ahead, the most economic method of reinforcement was at a voltage of 33 kV. Extending the period to fifteen years, however, with a consequent increase in load, was sufficient to enable 132 kV reinforcement to be justified. It is realized that load forecasting up to 15 or 20 years ahead is difficult, and can eventually lead to excessive expenditure if the long-term loads are on an optimistic basis and do not materialize. In planning the transmission and sub-transmission networks, however, it does seem essential to look far enough ahead so as to avoid networks growing in a haphazard manner. As a safeguard against over-expenditure, the long-term plan should, if possible, lend itself to stage-by-stage development so that initial expenditure is low.

In Section 3.2 it is stated that voltage reduction from 66 kV is effected in two stages; first to 33 kV or 22 kV and then to 11 kV. It is, of course, appreciated that existing networks will affect the planning, but has consideration been given to direct transformation from 66 to 11 kV or even 132 to 11 kV? In one case in this area, where there is a heavy block of load of 30 MW due to some factories in a small area, the cheapest scheme so far has proved to be an injection from the 132 kV system down to 11 kV. It will, of course, mean a 250 MVA fault level on the distribution network. It is noted, however, that in some instances it has been necessary to depart from the 150 MVA standard to 250 MVA. Would the transformation from 66 to 11 kV or 132 to 11 kV be suitable in these areas, with a consequent saving in double transformation losses and capital costs?

In Section 3.5 the Leach system of 4–12 MVA transformers is mentioned. With this arrangement there is double-busbar switchgear on both sides of the step-down transformer, and on the l.v. side there are two section switches and two coupler switches. The reliability and flexibility built into this intake point is, in fact, a maximum, and no doubt was influenced by the importance of the load to be dealt with and the desire for security of supply. It is interesting to note, however, that a feature of the Board's plan is the extended use of feeder transformers, and in this respect I would mention that the centre of a large northern city will shortly be fed from a power station five miles away by means of 132 kV feeder transformers. There will be no switchgear on the h.v. side of the transformers at the receiving end, where the ultimate load which can be supplied will be approximately 160 MVA.

With regard to Section 4.1, with the type of major substation described by Leach and an adherence to a fault level of 150 MVA on the distribution network, it is understandable that continuous attendance was necessary in order to effect speedy restoration of supply in emergency. On economic grounds it is also understandable that the trend is towards more automatic features in the operation of the main transformers. Does this mean full supervisory control or, say, automatic switching? It is interesting to note that, assuming a fault level on the 33 or 22 kV busbars of 750 MVA, three 12·5 MVA transformers suitably reactored on the 11 kV side can be operated in parallel on to a 150 MVA 11 kV network. A fourth transformer standing by could be switched in automatically in the event of the loss of one of the three transformers. The firm capacity of such an arrangement (ignoring busbar faults, of course) would be 37·5 MVA continuously or 45 MVA on a cyclic rating basis.

On the other hand, if full supervisory control is used from the Valley Road Control Centre, it would be interesting to know how one could operate, say, four 15 MVA 33/11 kV transformers, bearing in mind the extension of the use of feeder transformers. Is the method used that which, incidentally, is used elsewhere, i.e. to run the four transformers radially, relying on the facilities for rapid closing of the section switches from the remote-control centre in the event of a fault?

If so, it does, of course, involve a loss of supply to the consumer fed from the faulty transformer, whereas the automatic switching arrangement does not. The best and most economic method of meeting a block of load approaching 40 MVA at 11 or 6·6 kV and at a fault level of 150 MVA is one which is of interest to many distribution engineers, and more detailed comments on the latest approach to the problem would be appreciated.

**Mr. J. E. Peters** (*at Manchester*): From the point of view of electrical loading, number of consumers, growth of load and annual expenditure on new capital works, the London Electricity Board and the North Western Electricity Board compare very closely; yet whereas the North Western Electricity Board operates over an area of over 5 000 square miles, the activities of the London Board are concentrated into one-twentieth of that area, namely 257 square miles. One would therefore expect to find some major differences in distribution technique to deal with such a concentrated load, and examples of this, such as the two schemes for 'solid' medium-voltage networks, have been referred

to in the paper. Nevertheless, there is much in common in the general pattern of distribution, and I was very pleased to note the author's reference to the substantial savings which had been achieved on transmission within the London area, by what I should like to call the new planning freedom given to supply engineers by the 1947 Act.

In spite of the many worthy achievements of the 1926 Act, it did encourage inefficient planning by permitting energy to be bought on more favourable terms at the busbars of 'selected' generating stations. Extensive transmission systems were, in consequence, frequently adopted by local undertakings in order to obtain the benefit of these reduced tariffs.

Since nationalization the introduction of a standard tariff for energy taken at any C.E.A. supply point, together with the obliteration of former undertakings' boundaries and the agreement to adopt the most economic scheme of reinforcement, irrespective of the division of capital expenditure between Central Authority and Area Boards, has given supply engineers a great opportunity to plan system development on a much broader scale and on a more rational and economical basis.

From my experience in the north-west, I would give as a conservative estimate that a 30% saving in capital expenditure has been achieved, and that, for the capital expended, we have built up a more efficient network having reduced losses and greater load-carrying capacity.

I share with Mr. Wain concern as to whether the industry is looking far enough ahead in its forward planning. If the load doubles in the next 10–15 years—and there is every prospect of this happening—what are the Area Board networks going to look like in 1970, and where are the substation sites coming from in our congested and built-up areas? We may well have good reason to be thankful for the large d.c. rotary-convertor substations built in the past, for in many cases these will form admirable sites for future 33 kV substations.

I would make a plea for a longer-term financial programme. For the second time in our short history, Area Boards are being asked by the Government of the day to reduce their rate of capital expenditure. Our already limited resources in engineering man-power are being wasted in reviewing capital programmes to see which schemes can be deferred with the minimum of risk to security of supply.

With regard to Section 5.1, all Area Boards are young in the experience of administration and the organization of a very large undertaking; we were virtually created overnight and have not had the benefit of growing slowly. It is inevitable, therefore, that, as time goes on, certain adjustments may be found necessary. Specialization in a particular function seems to be unavoidable in our organization, and many of us are searching for a means to remedy this so as to provide our younger engineers with a wider experience. I was particularly interested, therefore, to read of the experiment being carried out by the London Electricity Board, and should be glad if the author would explain in greater detail how the Board manages to eliminate functional working, as far as possible, in the enlarged districts.

**Mr. F. Linley** (*at Manchester*): The author stated that Manchester was supplied by one authority, and inferred that the integration problem was therefore very small compared with the London problem. This is not quite the case, as if a comparable area to that of the London Electricity Board is superimposed on Manchester it contains 20 supply authorities. Whilst the problem was not as great as that of the London Electricity Board, nevertheless it was a major one.

The film of the cartridge method of spiking a high-voltage cable was very interesting, but the fundamental requirement is to select the correct cable every time, and in the Manchester Sub-Area the instruction is enforced that no high-voltage cable shall be spiked or cut into until a positive (not a negative) identification has been made. I should be glad if the identification methods used in London could be described.

The problem of integrating the operations on the 40 different high-voltage systems must have been a major one, and I should be glad to know how far this has progressed and how nearly the Area high-voltage systems are controlled as a single unit.

Knowledge of the extent of the use of supervisory remote switching for this purpose would be of interest. Such a system can often be justified on purely economic grounds, but I feel that the great increase in traffic congestion, which makes the use of mobile operating engineers a very slow method, is another important factor in the case. At a public inquiry on the Manchester traffic problem, it was authoritatively stated that, at rush-hour periods, the average speed was 5 m.p.h. Clearly if a fault occurs in the city during these times, the problem of getting operators to the site can introduce major delays in the restoration of supplies.

The author mentioned that the Board anticipate having to use compulsory powers more frequently in the future in obtaining substation sites. This is indeed a major problem in Manchester, and, as a previous speaker suggested, the existence of ex-rotary-substation buildings may be of considerable help in the siting of future 33 kV to 11 or 6 kV substations, but I am even more concerned about the problem of obtaining the more numerous 11 or 6 kV to low-voltage substations. It may well be that the underground chambers containing the Wordingham pillars may offer a solution to this problem.

The author also mentioned that the Board 'admitted' the use of the sheathing of the supply-cable network for the purpose of forming an earth connection for consumers' installations. I should be glad to have this point amplified and to know whether the London Board bring out in each consumer's premises a connection from the cable sheath and invite the consumer to use it, or whether they merely permit the consumer to take all the necessary steps he may desire.

**Mr. R. H. A. Reid** (*at Newcastle upon Tyne*): The paper is very difficult to discuss, since the London area has a load-density problem which makes it different from any other area, and thus one can only make comparisons.

In the early days London had the benefit of the advice of S. Z. de Ferranti, while in the north-east we benefited from the outstanding ability of Charles Merz and André Reyrolle. All three were pioneers in the development of the distribution of electricity as it is at present. That they had different ideas on the method to be adopted—one supporting a radial h.v. system with a ring l.v. system and the others supporting the ring h.v. system with radial l.v. system—provides interest, and although I do not think that one method is, in general, superior to the other, each has its use. The questions of cost and reliability have to be balanced against each other.

In this area the ring h.v. system is used, and 66 kV is considered as a distribution voltage, as 20 kV has been for many years past. I note that in London these voltages are considered to be transmission voltages. The necessity arose through the long distances of transmission with large bulk loads remote from power stations, and the restrictions on building power stations where they were needed to meet those loads. Our problems are therefore different, since greater reliability in h.v. supplies is required. In the cities and towns this pattern is continued, except in new housing estates, where a change is being made to a radial h.v. system from one main substation on the ring.

As in London, the North Eastern Electricity Board has also decided on the use of 500 kVA substations as a standard, following investigation into the most economic l.v. network layout. The experience of the London Board, however, would prove

useful in regard to the method of obtaining substation sites in congested areas in the towns.

The voltage up-rating of the 6 kV cables in London is of great interest, as the limited experience has been unsatisfactory in this area, although a special 11 kV cable has been used on the 20 kV system in the north-east for many years.

From the London standard-service particulars, it would appear that London networks are of the 3-phase type. Although this is the case here to a certain extent on the outskirts, the main city networks are at present of the single-phase 3-wire type—a relic of the old d.c. system.

As regards administration, the operation and maintenance of equipment at all voltages is carried out by the District engineering staffs, except for protection and telecommunication, which are supervised by specialist staffs at Area Headquarters. Otherwise the arrangements are similar in both areas.

The incidence of faults on the h.v. system is greater in the north-east, and is probably aggravated by ground subsidence owing to mining in the area. On the 66 kV system the fault incidence averages about one per 100 miles per annum on old solid-type cables; oil-filled and gas-pressure cables are almost free from electrical failures. On the 20, 11 and 6 kV systems the rate is approximately three per 100 miles per annum.

**Mr. F. E. Heppenstall** (*at Newcastle upon Tyne*): The film showing the operation of a cable spiking gun indicated a very small external disturbance when used on a live cable, and it appears to show that the protective gear had a very short time of operation. Would the author state the type of protection in use?

The author refers to the necessity of changing the organization of the Sub-Areas and Districts to give engineers greater distribution experience. The division of work in a large organization is always a problem, which, I feel, some of the Electricity Boards have not solved in a satisfactory manner, from the point of view of maintaining the interest of the engineer in his work, or giving him the opportunity to widen his experience. I do not see why it should be necessary for a man to confine his activities to one small subject over a large area, when surely he could better be employed in attending to a number of subjects in a smaller area.

The solid distribution system referred to by the author, and previously described by Mr. Leach, is an interesting subject which could well provide material for a paper in itself. As the London Electricity Board has a number of distribution systems, I would like to see comparisons between the different forms of network, i.e. solid against radial, with comparisons of the relative value of the service provided and cost of maintenance. Does the solid system make for difficulties in fault location, and do the $0 \cdot 007\,in^2$ service cables burn away if faulty?

The London Electricity Board has successfully up-rated 5 and 6 kV cables to 11 kV in the circumstances described, and this is a method which might well be adopted elsewhere for increasing the load capacity of h.v. networks.

The standardization of supplies in the London area is evidently a serious problem, and at the present rate it appears it will take nearly 25 years to complete. It should be remembered that, with the increase in prices, which is now common, and the fact that in change-over work it is always the easier jobs that are done first, a rigid programme is necessary if the standardization is to be completed in any reasonable period.

**Mr. N. Care** (*at Cardiff*): I was very gratified to see the author's film on the use of the cable spiking gun in London, as this was an application developed many years ago in the Birmingham Corporation Electric Supply Department's area.

With regard to the planning of distribution networks in the London area, I realize that many advantages might result from the limitation of the short-circuit level of 11 kV networks to 150 MVA, but, in my experience, this may well result in other disadvantages, such as the limitation of the size of step-down power transformers from the 132 kV or 66 kV systems, and the sectionalization of 33 and 11 kV systems by the use of radial feeders, or ring mains open at one point. Now that manufacturers are generally able to put forward very economical 11 kV units of 250 MVA rating, is the policy of dead-end supplies still correct, or would improved continuity of supply result from the use of parallel feeders and closed ring mains with automatic protection?

It is noted that 33 kV substations are generally unattended, and therefore any fault on an incoming 33 kV cable might result in a temporary interruption of supply to consumers until switching could be carried out. Are means provided for the automatic transference of load from faulty to healthy feeders, in order to cover such conditions?

The use of the solid system of distribution is interesting, and in my opinion can be applied where the load densities are less than those given for Central London. I have found that the real limitation with this system has been the provision of supplies to large stores and blocks of offices, where a single transformer is fully allocated to the load in that particular block. In these cases the simple protected ring-main system, with the circuit-breaker in the ring and the transformer controlled by an oil switch, might be a better proposition. It is realized that faults on m.v. underground cables might be self-sealing, but I should like the author's view on the protection of small service cables fed from solid networks, where a fault, say in consumers' premises or on the service termination, might not be so easy to control.

With the load development expected in the London Area it seems to me that problems will arise in providing sufficient space for the 11 kV copper necessary. Would it not be necessary to transfer even more load directly to the 33 kV system and take 33 kV substations nearer to the load centres? It is noted that the author recommends outdoor installation for 11 kV substations, and now that 33 kV switchgear of the metalclad type is available for outdoor use, it might be possible to provide a very compact 33/11 kV substation arrangement for outdoor installation, and so limit the distance over which the load has to be carried on the 11 kV underground cable system.

**Mr. G. H. Bowden** (*at Cardiff*): Is it now economic in the London area to take up more direct laid cable than previously when the price of copper was very much less than that now obtaining? I should imagine that the scrap value of heavy copper cable would justify serious consideration being given to the abandonment of much of the non-standard cable work.

With the development of the technique for jointing aluminium conductors, it seems that the new conductor material will, in the face of world-market conditions for copper, require to be kept closely in mind when planning future extensions. It would be interesting to know whether, on low-voltage networks, the use of aluminium conductors, with lead sheaths and the usual armouring, would exhibit the same burn-off characteristics as are shown by existing cables.

It is gratifying that the author has made clear that inspection and maintenance of isolators attached to network transformers is provided for by allowing one feeder in six to be taken out of commission, by relying on network intercommunication to maintain supplies. Otherwise this design of substation might be used in conditions where proper maintenance would not be possible. I am glad that the designer of the cable connection shows an appreciation of the avoidance of heavy forces that may result from the expansion of cable conductors being applied to transformer terminal bushings.

I believe that a large number of faults on the London low-voltage networks occur in cable boxes, and the number installed

therefore gives an indication of the reliability of supply that can be expected from any distributor.

In general, I believe that little trouble has been experienced with electrolytic corrosion, but the author's views on the usefulness of cathodic protection would be of interest. With the heavy fault currents that can now flow in this concentrated network, armouring resistance should become of increasing importance. Is it considered that, in these networks, the armour clamp of the usual network tee box is entirely satisfactory when fault currents may be diverted into neighbouring communication cables at points where high resistances are present in the run of cable armouring?

The switchgear on the 6·6 kV system at Hackney, which, I believe, was installed about 25 years ago, required extremely heavy busbars and also heavy current switches that set a problem for the manufacturer concerned. The busbars were made of hollow copper castings, the interior being filled with cement. The switches for the currents to be handled had heavy throw-off forces, and redesign of the manufacturers' largest solenoid closing gear was necessary. I think that this case must represent the absolute limit of concentration for any 6·6 kV system, and it would be interesting to know what maximum grouped capacity at 6·6 or 11 kV is considered good practice in planning extensions.

**Mr. T. Pickles** (*at Hove*): The London Electricity Board's extended use of 22 and 33 kV transformer feeders conforms to present-day trends. Where the reinforcement of sub-transmission networks is required, the exercise of ingenuity in the application of switchgear can result in considerable economy. On the subject of sub-transmission in large heavily-loaded areas, I think we shall find it economic to dispense with one of the stages of transformation between the transmission system and the m.v. distribution system, and I suggest that 22 or 33 kV will be unnecessary.

The author's words of caution about standardization are very wise. No doubt it offers economies by bulk purchase and reduction of stores; but it also brings the danger of mental stagnation. The author's choice of an example—the standard transformer size of 500 kVA—could hardly be less controversial in view of its widespread acceptance. There is nothing wrong with the 500 kVA size now, but when it is no longer the best, mental paralysis may prevent a change.

**Mr. A. Abbott** (*at Hove*): Electricity supply in London, having commenced on a parochial basis, resulted in some rather unorthodox electrical-engineering projects.

The extensive 66 kV cable network stretching some 18 miles from central London power stations to supply the then M.E.T.E.S. Co. was a *quid pro quo* arrangement. Such stations as that at Battersea were guaranteed a high load factor, the M.E.T.E.S. Co. receiving supply at preferential rates from the then London Power Co., rather at the expense of other consumers in closer proximity.

This 66 kV network has now become more of a liability than an asset, since the attendant problems of power station and load sharing, etc., in using it for feeding into, instead of from, London are causing the supply industry an engineering and economic headache.

I trust that full use will be made of 132 kV bulk supply into central London, and not 66 kV, which is a somewhat non-standard voltage, and step down to 33 kV for general distribution. Suitably designed indoor switchgear is now on the market, and bulk supply points up to 200 MVA at this voltage could be sited in densely populated areas.

The author mentions a present aggregate generating capacity in London of 3 250 MW; it might be difficult, owing to the lack of further sites, etc., to exceed a figure of 4 500 MW. When present generating plant becomes time expired with large amounts of load at 33 kV in close proximity to stations, as for instance in London, there may be an economical case for replacing it with 33 kV generation, thus reducing switchgear costs, etc.

**Mr. W. E. Gibbs** (*at Hove*): The Electric Lighting Act of 1882 did not permit the interconnection of power stations, probably because the protective equipment of the day afforded inadequate safeguards should a failure occur in one station or another. I think that the first permitted interconnection between two power stations, at any rate in London, was effected when the County of London Company's station at Wandsworth was interconnected at 11 kV with that of City Road just before the First World War.

Much has been said with regard to the development of the 33 and 66 kV supplies in London, and reference has been made to the economies which were effected by supplying the new substation at Bengeworth Road from Deptford East instead of from another source eight miles to the east, which would have resulted had the pre-vesting day arrangements prevailed. The author has not pointed out, however, that certain organizations possessing large power stations and operating under Section 13 of the 1926 Act could transmit comparatively large blocks of power over long distances more economically than would be the case if they were to take supplies to the same points at the then ruling Grid tariff. It was for this reason that the supply to Bengeworth Road, which was planned before vesting day, would have been taken from the Barking power station.

The author states that distribution systems are being designed with sufficient reactance to restrict the general fault levels in the 11 kV distribution system to 150 MVA, and I strongly support this. It is true that, in certain cases, systems have to be designed with a fault level of 250 MVA, but I would like to regard these as exceptional. This will enable hand-operated gear to be employed, and should be of considerable benefit in those districts which are being developed on 11 kV lines.

Could the author supply information on the average fault level on the radially interleaved system?

**Mr. E. A. Pannett** (*at Hove*): Has a standard system of street-lighting control been adopted by the London Electricity Board? If 5-core l.v. distributors are used, where are the control points, and what is the extent of the area controlled from each?

**Mr. G. O. Penfold** (*at Hove*): Considerable attention is being given to the creation of smokeless zones. This will be reflected in the maximum demand created in the future. Has the London Electricity Board in mind the adoption of any method to control peak demand?

**Mr. R. E. C. Maundrell** (*at Hove*): There are half a million prepayment meters in London, and in conjunction with the d.c. to a.c. change-over programme or otherwise, is there any intention to reduce the number in favour of credit supplies?

With regard to the change of consumer without disconnection of supply and the fact that 'In general, no test need be made in such cases unless inspection suggests the special desirability of testing', the proviso 'unless inspection suggests' implies that some degree of inspection is carried out, but would this be other than by a meter reader? Even so, considerable discussion has arisen on the known deteriorated condition of many installations and the deadlock reported on the legal liability of the landlord or tenants to put such installations in a safer condition and the absence of any credit terms for rewiring. Is this position likely to be resolved? Has the use of storage block heaters or other systems resulted in raising the late night to early morning dip in the load curve to any substantial degree?

**Mr. A. J. Bode** (*at Edinburgh*): Having regard to the future demand in London, which gives a load density of 200 MW per square mile, my estimate of the requirements, allowing for 20% spare capacity, is as follows:

| *Per square mile* | |
|---|---|
| One generating station | 240 MW |
| Four main substations | $4 \times 60$ MVA |
| 500 transformer chambers | 500 kVA each |

Taking into account these requirements, the original standards laid down by Leach (see Reference 4 of the paper) are probably inadequate. For instance, switchgear on the 11 kV system should be increased to 250 MVA.

The size of distribution transformers, if increased to 750 kVA, would considerably reduce the number of transformer chambers required and therefore the possible need for compulsory purchase. I note that it has already been decided to increase the capacity of the main substation transformers to 15 MVA.

Webb's system referred to in the discussion on the paper by Leach is less costly than Leach's scheme, and I wonder that it did not receive earlier consideration. Originally the primary objection appeared to be the solid connection of a very extensive Grid system without adequate knowledge of the behaviour of l.v. faults.

Are the Board's engineers now satisfied that l.v. faults burn clear without undue disturbance to the system? Transformers embodying the h.v. and l.v. switchgear were also referred to by Mr. Webb during the discussion on the papers by Leach and McLean (References 4 and 5 of the paper). I am surprised that this arrangement has not gained greater popularity, since it is virtually a substation in itself and could be used in a slightly modified form on other networks. The arrangement shows a reduction in installation time and cost against the conventional substation with three items of equipment. Is the isolator suitable for fault making and breaking?

Is the method of security of tenure for transformer chambers by leasehold, freehold or supply agreement?

The two-tier organization as replaced the three-tier organization, and it would be interesting to know the advantages to be gained from this.

**Mr. F. S. Kynoch** (*at Edinburgh*): We must assume that the radial system has proved operationally satisfactory, and it would be interesting to have some figures giving the number of faults and interruptions of supply per section per annum.

In the case of l.v. faults burning clear, has this resulted in many instances of pavements being ruptured?

Taking into account the rate of growth of maximum demand, the only economical method of supplying a city is by distributed transformers, thus keeping the amount of l.v. copper at a minimum, since this is one of the most expensive items in distribution. In addition, the improved voltage regulation sells additional power.

A large number of transformer chambers require the capital cost per chamber to be kept to the absolute minimum. This may mean cutting one's losses and abandoning a system of unit protection on the h.v. systems, since the cost of protective gear per mile of cable would become prohibitive. On a unit-protected underground system, experience has shown that more switches are tripped accidentally than by any other means.

In Section 3.5 transformer protection is stated to be by over-current and earth leakage on the h.v. side and by over-current on the l.v. side. What percentage on the l.v. winding is covered, and would it not be better to use a reverse-power relay on the l.v. side as being more sensitive and possibly able to clear an h.v. cable earth fault which has operated the earth-leakage protection at the main station? As transformers age the fire risk is going to increase. Will the stage be reached when it will be advisable to install fire protection? In the interleaved system a large number of transformers are switched off until an h.v. cable fault is repaired. At the expense of a little extra time to hand-close the circuit-breakers, the automatic reclosing feature could be omitted. This system would seem to be lavish in the use of h.v. cable in its early stages, and to suffer from a lack of switching facilities, thus making it difficult to operate unless a large surplus of transformer capacity is always available. In both systems, has it been proved that all l.v. faults burn clear without excessive damage to cables?

In Section 4.1, should not a larger size of transformer be considered rather than duplicate transformers and l.v. switchgear when sites are suitable?

In Section 4.2, a visual inspection of consumers' installations after less than three months' disconnection is specified. Is this sufficient, since many faults could escape notice, and would not the Board be involved in any subsequent claim by the consumer because of a faulty installation?

**Mr. J. Wainwright** (*communicated*): The information in Section 3.3 is supported by experience on one Swiss cable system* where cables installed to work at 10 kV were up-rated to 16 kV after 30 years in service. In this case a considerable amount of experimental work was undertaken involving long- and short-term tests with alternating, direct and impulse voltages.

Such time- and material-consuming tests may not be justified economically, and the most satisfactory procedure (provided that the loss-angle/voltage curve is acceptable) may be to increase the working voltage and await results. It would be interesting to have the author's views on this, together with details of any tests which he may have made.

On the general subject of insulation testing, can the author give brief details of the maintenance procedures used on the London system? Most systems pay close attention to the state of insulating oils, but in this country at least, routine testing of installed plant *in situ* appears to be exceptional. What tests, apart from the ubiquitous insulation-resistance test, are performed regularly on the other items of equipment?

The capacitance of the cable system is so large that, at certain times, a great deal of generating plant must be run merely to supply the charging current. Are not reactors installed to improve this situation?

Section 3.2 includes brief details of the substation transformers, but no information is given on the distribution transformers. Presumably these units are of the ON type. Have any 'dry' type units been installed, and is any consideration being given to units with Class H insulation? In America, transformers cooled by chlorinated aromatic liquids have been in use for many years, and there is a definite trend towards high-temperature 'dry'-type units for mounting in chambers beneath the 'sidewalk'.

**Mr. D. B. Irving** (*in reply*): The interest shown in a paper confined to London's electricity supply is gratifying, and I thank the various speakers for the items of historic interest which they have brought out.

In replying, it is convenient in the space available to deal generally with the technical matters rather than with individual contributions.

*Interconnected Networks*.—The proof of interconnected l.v. networks lies not in calculated niceties of cable design, copper or aluminium, laid direct or drawn into ducts, nor in precise values of fault effort, transformer size or voltage, but simply on the observed fact that, if given a small time delay in which to do so, l.v. faults are self-clearing. Twenty years of operating experience in London have shown a far better performance than with other types of system, both in voltage regulation and in freedom from interruption of supplies to the consumer.

The voltage control required is very largely that needed to take account of incoming supply variations. In one pre-vesting undertaking a range of $\pm 5\%$ was found to be adequate.

Faults, when they occur, often stem from mechanical damage

* Schilling, E.: *Bulletin Schweizerischer Elektrotechnischer Verein*, 1954, 45, p. 169.

and are located by external evidence. Explosions are nearly always attributable to the older systems, particularly d.c., and are associated with gas, either independently supplied or internally generated as in v.b. cables. The typical short-circuit level is 25 MVA. This may be exceeded in certain very dense sections, but at the higher fault levels the significance of the fault impedance itself is greater.

The nominal transformer size of 500 kVA is both the most convenient and the most economical over the typical range of load and street densities in London. At the highest densities where there may be a theoretical case for larger units, this would call for a much more expensive and heavier l.v. network. In practice, the load tends to be concentrated in individual buildings, and such cases require individual consideration depending on the precise load characteristics.

Dry-type or non-inflammable liquid-filled transformers are not used as standard because of expense. Transformer chambers are leased or licensed, and their acquisition is an important part of the district engineer's task.

Experience in the acquisition of sites for the wholly outdoor unit has not so far been encouraging. There is quite strong prejudice against them on so-called amenity grounds, and enclosure in buildings is often unavoidable.

*Cables.*—The decision to re-grade cables from 6·6 kV to 11 kV has been based on past history and knowledge of cable construction rather than on any special tests. Joints have sometimes been re-made.

All available methods of cable identification are used, including current injection and search coil, and cable spiking is always used where any doubt remains. The cable shown in the film was protected by Beard–Hunter protection.

Despite the high value of copper, labour and reinstatement costs militate against the recovery of many miles of derelict cable in London.

*Substations.*—The longest possible view is taken in planning new intake points, but the predominant factor is nearly always the existence of adjacent sources of supply at established voltages which may be neither standard nor the most convenient. Since vesting day, 50 MVA substations based on 22/6·6, 22/11, 33/6·6, 33/11, 66/11 and 132/11 kV transformation have all been used. The trend is towards fuller use of supervisory control to avoid increasing the attendant staff. Large transformers are provided with Buchholz protection, but this is not regarded as practicable or economic on the distribution transformers.

The target fault effort is 150 MVA in the h.v. distribution system. Were it not for extensive existing equipment, 250 MVA would be acceptable. Particular attention is paid to the adequacy of earthing connections and the ability of earthing switches to make the short-circuit currents.

*Consumers' Installations.*—The practice on the distribution network is to bond all cable sheaths and armour throughout so as to form as effective an earth grid as possible. The Board do not offer to relieve the consumer of his obligation to provide an effective earth for his own installation, but are prepared, if local system conditions permit, and sometimes subject to a suitable indemnity, to agree to the connection of the consumer's earth to that of the Board.

Where a change of consumer takes place without disconnection of supply, a detailed inspection is not made. Indeed, with prepayment meters the Board may not be informed of changes of tenancy. The policy is to discourage the use of prepayment meters. Experiments in outdoor mounting of meters are being tried.

In new buildings, direct electrical floor heating, with off-peak supply, taking advantage of the storage capacity of the building structure, is extending.

*General.*—Whatever form an Area Board organization may ultimately take, the same functions have to be performed and the same regions have to be controlled. With the two-tier organization the men undoubtedly feel nearer to the management and the managers are nearer to the men. These benefits are already showing to advantage in the four new managed districts which it has so far been possible to set up.

Operation of the transmission system, and those other functions which can only be properly performed over the Board Area as a whole, will remain at Headquarters.

# DISCUSSION ON

# 'PROBLEMS OF HYDRO-ELECTRIC DESIGN IN MIXED THERMAL–HYDRO-ELECTRIC SYSTEMS'*

*Before the* North-Eastern Centre *at* Newcastle upon Tyne *24th October*, 1955, *the* Mersey and North Wales Centre *at* Liverpool *5th March, and the* South Midland Centre *at* Birmingham *7th May*, 1956.

**Mr. E. C. Scott** (*at Newcastle upon Tyne*): The authors indicate the importance of the characteristics of the load curve in determining the design of a low-load-factor hydro-electric project. The interpretation of these characteristics, however, is to a large extent a matter of opinion, and there is generally some conflict between those responsible for the design and for the ultimate operation of the project. As the authors point out, the annual load/duration curve cannot reasonably be used, since this would mean that the hydro-electric plant was required to meet only the extreme winter peak demands. The adjusted load/duration curve provides a more reasonable basis where, presumably, the hydro-electric plant would meet all demands above a predetermined magnitude for the time of year. Such an arrangement would require a more accurate technique of load estimation than is now, or will ever be, practicable. The operating engineer would like the hydro-electric plant to meet its fair share of the peak demand every day. Where the design and operating engineers are working for the same authority, a satisfactory compromise solution can usually be found; but where they are working for separate authorities and a tariff has to be agreed, considerable difficulty is found in reaching the solution.

Any alteration in the characteristics of the load curve would have an influence on the value of low-load-factor hydro-electric schemes, and it is important to consider whether there is any real prospect of such a change.

The extensive load shedding of a few years ago drew attention to the peak-load problem, but the solution which received most publicity was the provision of more generating plant to meet the peak demands. Much thought has, however, been given to what may be described as 'painless' load shedding—the disconnection of load such as thermal storage heaters and water heaters which could be done without inconvenience to the consumer. The magnitude of such load is of the same order as the capacity of hydro-electric plant which could be developed in this country. One of the advantages of 'painless' load shedding is that it relieves the peak-load problem of the distribution system as well as that of the generating plant. 'Painful' load shedding had a very serious effect on the value of low load factor hydro-electric plant and the 'painless' variety would have an equally serious effect. The financial justification of the cost of load-control equipment, however, is just as complicated as that of low-load-factor hydro-electric projects. The characteristics of the load curve would also be substantially changed if shift working were adopted by industry.

**Mr. W. Brittlebank** (*at Newcastle upon Tyne*): In a mixed system in which hydro-electric plant operates on a low load factor, the normal system operation is to use the thermal plant for base load and the hydro-electric plant for peak load. Since the week-day peak, particularly the morning one, comes on very rapidly in a highly industrial area, it is of paramount importance that the peak load plant is brought on-load as quickly as possible. The need for minimizing the running costs of hydro-electric plant has led to the introduction of group control centres in which all stations in the group, which normally covers a catchment area, are remotely controlled. From the system-operation aspect this is most important, for by providing indications of water level it enables the control engineer to make the most efficient use of the available resources. Moreover, by providing fully-automatic, rather than semi-automatic, control it is possible for a single operator to start, synchronize and load any number of the machines in a matter of minutes, for after the initiation of the sequence the subsequent control is completely automatic and proceeds simultaneously on each machine.

The difference between semi-automatic and fully-automatic control is that with the former the starting sequence runs up the machine to approximately synchronous speed, but the synchronizing is by remote control from the group control room. Fully-automatic control synchronizes the machine automatically and can, if required, load it to any given value. There is, of course, a limitation to semi-automatic control because synchronizing is not possible at distances exceeding 12–15 miles.

It is sometimes the case that in a group of hydro-electric stations the major station would operate on base load. In Section 6.4 it is suggested that base-load hydro-electric plants will continue to be staffed. The major station in a group is often the ideal position for the group-control centre from both the personnel and technical considerations; semi-automatic control would be the most economic for the base-load station, which would be staffed by the group control engineers and maintenance staff rather than by the normal power-station operations staff, bearing in mind that the remainder of the stations in the group would be operated by light-current technique. For major base-load plants which do not form part of a group, semi-automatic control is often more economic, because no power-station operators would be required and a more compact arrangement of control is possible.

**Mr. J. Warnock** (*at Liverpool*): The larger bulges on the upper part of the adjusted load/duration curves were probably caused by the use of part-time-loaded steam plant which could not be run to meet the exact requirements of the loading curve. With the nicety of control possible with hydro-electric peak-load plant, a closer adherence to the requirements of the load curve is possible.

The inclusion of one additional pumping unit in a normal hydro-electric plant fed from storage is a worth-while improvement; by using off-peak pumping energy, the storage can be topped up as required and additional energy generated off the higher pressure head thus made available from the reservoir.

The proposed pumped-storage scheme at Ffestiniog in North Wales will operate on an average head of 1 000 ft between the upper and lower reservoirs. Four 75 MW vertical-shaft units will be installed, the generator/motor being at station floor level with the turbine below the generator. The pumps will be below the turbine and connected through an oil-operated toothed coupling on the vertical shaft, which is engaged or disengaged when the machine is at a standstill.

* Haldane, T. G. N., and Blackstone, P. L., Paper No. 1749 S, November, 1954 (see 102 A, p. 311).

The new plant will be connected by 275 kV overhead transmission to Connah's Quay power station, some 40 miles distant, and thence to the Manchester area over a similar distance to connect into the main Grid system. The maximum generating time proposed for the plant is to be four hours daily, and the pumping time is expected to be about seven hours daily.

**Mr. D. F. Grant** (*at Liverpool*): The installation of pumped-storage plant for low-load-factor operation to meet a general increase in maximum demand entails a corresponding increase in the operating load factor of the associated thermal plant. This is not necessarily an advantage, and if taken too far could cause serious difficulty in providing adequate outage for overhaul and repair purposes. The conditions under which the British system has been operating over the past ten years, with high average loading but with peak reduction by load shedding, have simulated those to be expected with a large proportion of pumped-storage plant. The associated difficulties have been very evident. The percentage of spare plant required for any thermal system increases with load factor, and for this reason I suggest that any pumped-storage plant should be of greater capacity than the thermal plant which it displaces.

Table 3 shows very clearly the dependence of a pumped-storage system for its economy on a considerable differential between the value of off-peak and on-peak energy. A marked improvement in average load factor could seriously modify the present position, and by making it necessary to supply pumping power from less-efficient plant, make the operation of even an existing pumped-storage plant completely uneconomic.

If pumped-storage plant is installed its capacity should be small in relation to the total output of the system, and even then it may merely replace for peak purposes usable and still reliable time-expired plant without sufficient saving to justify its capital cost. I cannot agree with the authors' statement that, if the capital cost of a pumped-storage plant is less than that of an equivalent-capacity thermal station, it will give an overall advantage. The acceptance of such an opinion at its face value would be extremely dangerous.

**Mr. J. E. Macfarlane** (*at Liverpool*): From the aspect of the load diagram, how are the hydro-electric stations brought in, is it possible to give an order of merit in the same way as a thermal station, and how are the transmission-line losses considered?

**Mr. F. W. Skelcher** (*at Birmingham*): With reference to the factors which operate against the theoretical optimum use of plant in practice, some figures taken from the C.E.A.'s annual load/duration curve for 1954–55 may be of interest. The load factor of the base-load thermal plant varied between stations from 49 to 93·5%, the average of this plant (which covered 12·8% of the maximum demand) being about 72%. There was thus a considerable amount of under-generation in this group. There is no doubt that initial difficulties with new plant are a big cause of the reduced availability of the base-load group. The adoption of unit construction for recent thermal plant may also be a factor which reduces availability.

Less generation than the optimum from the base-load plant resulted in over-generation, mainly from the plant in the region extending from 54 to 100% of installed capacity, and plant in this region of the load/duration curve generated approximately twice the theoretical optimum. These figures indicate the extent to which the factors of reduced availability of base-load plant and lack of flexibility of peak-load plant operate against the theoretical optimum use of plant in practice. The authors state that hydro-electric plant is especially suited to rapid load fluctuations. While this may be an asset, it is not an exceptional advantage, because the load fluctuations can be predicted and, with proper design, thermal plant can be prepared to follow. For example, in a steam station it is possible to start and load a 30 MW set to full output within 23 min after a 6-hour shut-down. I believe it possible with thermal plant to achieve a quality of operation in the peak region equal to that of hydro-electric plant, but the price to be paid is that due to banking and standby losses.

In this area we have plants now that are operating at load factors of 5·7 and 6·7 respectively, and in our future planning—with capital restrictions and the fact that replacement is not permissible—we plan for load factors even lower than these.

The economic advantage of pumped-storage schemes arises from the saving in capital cost. Table 3 shows the cost of such a scheme as about £42 per kilowatt, which compares with about £114 per kilowatt for hydro-electric plant. It seems apparent to me that there should be an introduction of pumped storage on as large a scale as possible in this country within the next decade or so, especially if hopes regarding the reducing cost of nuclear generation should be fulfilled.

**Dr. D. A. Bell** (*at Birmingham*): It is often implied that nuclear generating stations are so inflexible in operation that they must be operated at constant output and all but the 24-hour base load carried by other types of station. I do not see why this should be so, since the thermal capacities should not be many times greater than those of coal-fired plant, and I think it should be possible to programme nuclear stations to follow the main trends of load, e.g. between day and night conditions.

Studies on tidal schemes are being pursued in France,* and one factor which may be important is the development since the war of the *groupe bulbe*—a hydro-electric set with a horizontal shaft and the alternator enclosed in a pressurized vessel which is submerged in the water stream.

This is claimed to make better use of the low heads of water (as compared with the conventional vertical-shaft equipment). Since the wheel has variable-pitch blades, the set can be used for pumping as well as generating; and with an increase of dam height to allow pumped storage above high-water mark, it is in principle possible to use a tidal station to cover peak loads at any predetermined hour, regardless of the phase of the tide. However, the only likely tidal scheme in Britain is the Severn Barrage, and the quantity of silt carried down by the river may be a serious difficulty here.

**Dr. W. G. Thompson** (*at Birmingham*): I find the economic approach given in the paper both timely and important, because there is always a tendency to regard present conditions as continuing indefinitely and it is essential from time to time to take an objective view of the means of meeting our requirements for electrical energy. New methods of energy generation, the rising cost of coal and the question whether we will have enough miners in a few years' time to maintain the coal output required are important factors underlying the economic considerations of the paper.

So far as the future is concerned, the die has already been cast by the fact that only about 30% of our present population are under 21, and in 10–15 years' time our growth-of-population curve will flatten at about 50 000 000; but the average age will be much higher than at present, so that the energetic younger man-power to perform the arduous tasks such as coal-mining will not be available, and it is therefore desirable properly to assess the economic importance of all methods likely to improve the supply of electrical energy.

One of the reasons why we have very few hydro-electric schemes in this country is that we have not the watersheds and storage reservoirs at high altitudes which would give the necessary falls in catchment areas to operate hydro-electric stations. There

* GIBRAT, M. R.: 'L'Energie des Marées', *Bulletin de la Société Française des Électriciens*, 1953, 3, p. 283.

are, however, a number of high valleys which conceivably could be dammed to provide the smaller quantities of water required for pumped-storage accommodation. I understand that one of the important factors in the economics of water-power storage is the amount of evaporation from the surface of the reservoir, because on a sunny and windy day one can lose the equivalent of several kilowatts as evaporation reduces the level of the reservoir. Has this point been borne in mind? Is there any geographical consideration as to the parts of the country in which to install pumped hydro-electric storage, and has loss by evaporation been taken into consideration in dealing with similar schemes oversea?

**Mr. H. M. Fricke** (*at Birmingham*): Is there a tendency for higher speeds in hydro-electric plant to enable higher outputs to be obtained with the same weights of material?

**Mr. E. V. Hardaker** (*at Birmingham*): Reference has been made to the possible deferment in the installation of steam plant through the introduction of hydro-electric plant. It may not be adequate to replace a certain thermal capacity with an equivalent hydro-electric capacity, since in countries where water power is available in any quantity, it does not always follow that this power is available at the time of peak load. Owing to this and to the fact that the amount of water is dependent upon weather conditions, a greater amount of hydro-electric plant may be necessary to give the same firm generating capacity as would be possible from a given capacity of steam plant. Have the authors taken this factor into account in comparing the capital costs?

**Mr. D. H. Tompsett** (*communicated*): In the design and operation of a thermal system, statistical methods are appropriate for studying the economics of reserve capacity and the reliability of supply as affected by forced outages. In a hydro-electric or mixed system the variable nature of the water flow is an additional factor which can be properly taken into account only by a statistical approach. For a general survey, such as that provided by the paper, it is not possible to consider system demand characteristics in any greater detail than that represented by typical load/duration curves. In any particular system, the degree to which the designer's hopes are realized depends critically upon correct action always being taken by the load-dispatching staff in the hour-to-hour operation. The optimum use of the available water must be determined in the light of the recent past (i.e. the known precipitation and the state of the reservoirs), the immediate future (i.e. the forecast of weather and corresponding loads), and the probable course of events in the more distant future (i.e. the statistical trends of weather, load and availability of plant). Any attempt to find a rigorous solution to the problem would also need to include such factors as the relationship between the operating head of water and the efficiency of hydraulic plant, the transit time of water between plants located serially on a river and the transmission losses in the lines. It is clearly impossible that by ordinary methods this complex calculation can be more than approximately solved on the necessary continuous basis. Since the paper was written, however, some progress has been made in developing methods of solution to such problems by the use of high-speed digital computers.

In the paper it is several times stated that somewhat elaborate economic studies may be required in the design stage of a system; it is also implied that various factors may preclude the possibility of obtaining a precise solution. Many of the limitations may be of a fundamental nature, but those which previously arose from the necessity of performing repeated calculations can now be overcome. The general pattern for ideal operation on, perhaps an annual basis will, no doubt, need to be established by such studies as the authors describe. For the detailed investigation subsequently required in individual cases, and certainly for the actual optimization of daily operation, it would appear that high-speed computers could find economic application.

**Messrs. T. G. N. Haldane** and **P. L. Blackstone** (*in reply*): Uncertainty in the estimation of future demand, the shape of load curves, etc., must, as Mr. Scott points out, result in compromise decisions and the exercise of a good deal of judgment in settling the design load-factor of hydro-electric schemes. We fully agree that the difficulty of arriving at satisfactory decisions is greatly increased where two (or more) authorities are involved.

In reply to Mr. Grant, we feel that the difficulties he anticipates in regard to outage for overhaul and repair are not very likely to arise. In the first place the total amount of pumped-storage plant is unlikely to reach a large proportion of the maximum demand, and in the second it has to be borne in mind that hydro-electric plant is inherently more reliable than thermal plant. The facilities provided by pumped storage might, in fact actually ease the present situation in regard to overhaul and repair of thermal plant. For these reasons we do not think that it should be necessary for pumped-storage plant to be of greater capacity than thermal plant which it displaces.

We do, however, agree with Mr. Grant and Mr. Scott that an upward trend in the average system load-factor would tend to reduce the advantage of pumped storage. Our statement that pumped-storage plant is likely to have an overall advantage if it is cheaper in capital cost than new thermal plant relates, as we indicate in the paper, to long-term results and might not be true in the short term.

It is impossible to give a concise reply to Mr. MacFarlane's questions, since the proper use of a given hydro-electric station depends on several different factors, such as its designed load factor, the state of the storage reservoir, the anticipated daily demand and the anticipated rainfall. There may be other factors also to take into account, and in some instances the problem may be of a considerable complexity calling for the use of high-speed digital or other types of computer, as suggested by Mr. Thompsett.

With Mr. Brittlebank's comments on semi-automatic and full-automatic control we are much in agreement, as also with Mr. Warnock's suggestion that the inclusion of one additional pumping unit in a normal hydro-electric plant is a worth-while improvement. The addition of some pumping capacity should always be considered when physical conditions permit. This effectively gives an increase in storage capacity, and, in general the additional flexibility tends to increase the firm generating capacity—a matter referred to by Mr. Hardaker.

It is possible, as Dr. Bell thinks, that nuclear stations could be designed so as to be capable of load variation, but it would be most unprofitable to operate such stations at anything less than the maximum possible load factor, bearing in mind their anticipated low running costs.

Dr. Thompson is, we think, unduly concerned about evaporation from the surface of storage reservoirs. The total amount that can be lost by evaporation, particularly in a temperate climate, is likely to be a very small proportion of the total run-off or the total volume pumped.

In reply to Mr. Fricke, there is undoubtedly a tendency towards higher speeds in the design of hydro-electric plant.

# DISCUSSION ON 'GERMANIUM AND SILICON POWER RECTIFIERS'*

*Before the* RUGBY SUB-CENTRE *at* RUGBY *1st February*, NORTH-WESTERN CENTRE *at* MANCHESTER *7th February*, SOUTH-WEST SCOTLAND SUB-CENTRE *at* GLASGOW *29th February*, NORTH-EASTERN CENTRE *at* NEWCASTLE UPON TYNE *12th March*, NORTH STAFFORDSHIRE SUB-CENTRE *at* STAFFORD *4th May*, 1956.

**Dr. J. C. Read** (*at Rugby*): What is the effect of the hole-storage current on the power factor? If forward current is very suddenly applied through a germanium diode, what is the voltage drop through this as compared with its subsequent steady value? The results shown in Fig. 23 differ from those of Miller,† particularly for the higher concentrations of impurity centres, and it would be interesting to know the reason for the difference.

**Mr. G. R. Polgreen** (*at Manchester*): Most of the paper is devoted to germanium rather than silicon, which would now appear to play a major part in the future of semi-conductor applications, largely because silicon can operate at much higher working temperatures than any other type of metal rectifier. The same materials and techniques and the theory underlying them have also been responsible for revolutionary improvements in low-power rectification over a very wide range of frequencies in the development of new types of switch, photocell and refrigerator, and the recent transistor developments which will have a far-reaching effect on all branches of electronics. Already it is possible to see a new pattern in the future of telecommunication, computers, process instrumentation and control (automation), automatic telephony and in a wide range of military equipment.

The authors have devoted much attention to the comparison of germanium rectifiers with rotating machinery, but the most significant forerunners of this new type of rectifier have been those using copper-oxide and selenium plates. Having been engaged in some of the earliest work in this country on the latter types of rectifier in the 1920's and early 1930's, I remember the far-reaching impression then given by these rectifier units, which appeared to have no ageing characteristic and therefore gave possibilities of infinite life when operated at suitable ratings. At that time there were various alternative experimental devices, such as sulphide rectifiers, which had limited life although greater efficiency. Since those days the idea of a compact, silent, self-contained and inexpensive rectifier unit in a range of sizes from a fraction of a kilowatt to hundreds or even thousands of kilowatts has become commonplace and has greatly contributed to the convenience in the use of d.c. power for a large number of applications throughout industry and in the home.

One of the most important aspects of this subject is the availability of materials. The great advances in selenium rectifiers during recent years have been restricted by the shortage of the raw material, so that such large-scale uses as the supply of direct current from small alternators with associated rectifier equipments in mobile equipment has been greatly limited. Germanium is also a scarce material, but silicon is one of the most common materials in existence. Although the authors mention the very great problems in the chemical purification of silicon to eliminate the extremely minute impurities, in order to give the required electrical characteristics, considerable progress has been made since the paper was written and there would appear to be good prospects for the future.

There are various other performance advantages of silicon in comparison with germanium, but these are somewhat difficult to deduce from the paper because of the logarithmic scales used in Fig. 10(*a*) compared with the more conventional linear scales in Figs. 8 and 10(*b*).

The very great efficiency of these new rectifier elements is one of their most important advantages, and this shows up in practice in a great saving in the cost of electrical power, even on small units. This is an economic factor which needs to be considered in great detail by users of these rectifiers, because it will go far towards overcoming the difference in original cost even at the present stage of the low production and comparatively high price of the raw materials used for the rectifier elements. These high efficiencies would seem to render obsolete the commutator as means for generating d.c. power on any scale, and I should like the authors' comments on these points.

**Mr. J. P. McBreen** (*at Manchester*): I am interested in the application of germanium rectifiers to ratings of 100 kW and upwards, but the typical short-circuit requirement of eight times full load for half a cycle, given in Section 9.2.5, does not appear suitable for this type of duty. To achieve a fault-clearance time of half a cycle would necessitate a high-speed d.c. circuit-breaker, the cost of which would be prohibitive for most applications, assuming, of course, that the fuses shown in the parallel sections in Fig. 19 are there as protection against individual unit failure and not intended to rupture on system short-circuits. To limit the prospective fault current to eight times full load would require a transformer reactance of 12–15%, and this would make the direct-voltage regulation 10–12%. This may be acceptable for, say, electrolytic applications, but I doubt whether it would be for general industrial applications; in fact the normal 6% of mercury-arc rectifiers is sometimes unacceptable. I appreciate that induction regulators could be used, but these would increase the cost and lower the efficiency slightly. What is the free regulation and d.c. circuit-breaker clearance time for the 1 MW germanium rectifier shown in Fig. 19?

From Section 10.2.3 I imagine that difficulty may be encountered in matching the resistance of the parallel circuits in the larger current ratings, because of the low forward resistance of the units, and that the accuracy of this matching will determine the derating for parallel operation. How accurate must this matching be? Has any allowance to be made for unequal cooling in the larger ratings which employ several hundred units, or are special precautions taken to prevent this?

**Mr. F. Whyman** (*at Manchester*): I am interested in the application of rectifiers to the power circuits of electric locomotives and motor-coaches operating on single-phase 50 c/s supplies, and I hope that this type of rectifier will have a very wide application in this field. Unfortunately, germanium, with its low maximum operating temperature, would appear to have severe limitations for this application, but silicon would appear much more promising when it has been fully developed.

The paper gives a considerable amount of qualitative information, but gives very little quantitative information which would

* KINMAN, T. H., CARRICK, G. A., HIBBERD, R. G., and BLUNDELL, A. J.: Paper No. 1936 U, October, 1955 (see 103 A, p. 89).
† MILLER, S. L.: *Physical Review*, 1955, 99, p. 1234 (Fig. 6).

allow the potentialities for this application to be more carefully assessed, and I hope that the authors can supplement this.

For power circuits on a.c. railway vehicles the equivalent of a push-pull rectifier with smoothed d.c. traction-motor circuit would be used, and from the information given in the paper it would appear that the hole-storage effect, if present in anything like that magnitude indicated in Fig. 15, will be a very potent source of trouble, particularly as regards interference with telephones, radio, television and the power supply generally. A.C. electrifications will very often have the h.v. transmission line paralleling the railway, and very appreciable interference effects will be obtained.

So far as one can see, the hole-storage effect with the reasonable smoothing of the load circuit which is essential would add to the primary current waveform a very sharp current spike with a magnitude of 30–60% of the average current; this would very seriously aggravate the already difficult harmonic conditions and increase the difficulties of smoothing the load circuit. Condensers across the transformer secondary will minimize any voltage effects, but will aggravate the current waveform by further increasing the current spike, compensating later by a reduction below the normal current value. Will the authors give some quantitative information on this effect, and indicate whether there are any features in the physical design of the rectifier unit that will allow the effect to be minimized?

When mentioning the application of this kind of rectifier to traction, the authors indicate that a substantial saving in the transformer size would be possible compared with the multi-anode mercury-arc rectifier; but nobody would seriously consider such an application with the latter, the real alternative being single-anode mercury-arc rectifiers in bridge connection—with which exactly the same transformer economy would naturally be obtained. I suggest that the authors should not use this argument, for the semi-conductor rectifier should stand on its own merits rather than be assisted by an unreal comparison.

**Mr. A. A. Shepherd** (*at Manchester*): I should like the authors' views on the applications of silicon rectifiers in the power field. Although the advantages of their very low reverse current are obvious in communication, it would appear that this property is not so important in power rectification.

In the description of germanium rectifiers the authors discuss methods of rating the units based on junction-temperature measurements. What are the main factors which determine the maximum junction temperature allowable for a rectifier, and what factors limit the peak inverse voltage? It would appear that the resistivity of the semi-conductor is not the only factor of importance, and there seem to be differences between silicon and germanium in the correlation between resistivity and breakdown voltage.

In the alloy junctions described, the intersection of the junction with the semi-conductor surface is a place where any contamination would ruin the characteristic of the device. What precautions are taken to ensure that minute amounts of contamination are excluded from this region?

**Mr. R. H. Kelsall** (*at Manchester*): Germanium power rectifiers appear particularly suitable for use in the newer types of magnetic amplifier, where rectifiers having low reverse leakage are of particular importance in obtaining optimum performance. Have the authors considered them for this application?

The hole theory of rectifier action at junctions in semi-conductors is of particular interest. Is the same theory applicable to selenium and copper-oxide rectifiers?

Since the war we have been confronted with germanium and silicon as rectifier materials. Are there any other suitable elements in the periodic table which may show even more startling properties when used for rectifiers?

**Mr. E. Hanson** (*at Manchester*): At what frequencies does hole storage make the use of germanium and silicon power rectifiers impracticable?

What further progress has been made in the production of single-crystal silicon and high-power silicon rectifiers?

**Mr. O. L. Robson** (*at Glasgow*): The series-parallel connection referred to in Section 10.2.3 appears to require very serious consideration, although it is somewhat lightly dismissed by the authors.

I believe that the equipment shown in Fig. 17 is rated at 270 volts and 1 100 amp, and is built up from germanium units rated at 50 amp. From American literature I understand that the voltage rating of germanium units is about 65–80 volts, so that this equipment must consist of some hundreds of elements connected in series and parallel. The physical configuration of this mass of elements means that some will be considerably warmer than others, which will affect both their forward resistance and their reverse voltage-current characteristics. When this is allied to the variations in characteristics which are bound to occur in this large number of individual units, it would appear that considerable derating must be applied in respect to both current and reverse voltage when a number of units are connected in series-parallel. What is the relation between the apparent power rating of a single unit and the average rating of individual units in the equipment shown in Fig. 17?

**Mr. J. E. Davison** (*at Newcastle upon Tyne*): Since the announcement of the adoption of the 25 kV 50 c/s system for future railway electrification it has been generally agreed that the rectifier method of converting a.c. to d.c. supplies on electric locomotives and multiple-unit trains is the most preferable. The germanium rectifier appears an attractive device for this use, but raises a number of problems.

The critical nature of the operating temperature implies an efficient and reliable cooling system to avoid electrical breakdown, and in a multiple-unit electric train the heating cycle of the rectifier would follow closely that of the traction motors. On a typical all-stations service, e.g. in the London area, with an average of $\frac{1}{2}$–$\frac{3}{4}$ mile between stations, the maximum starting current is taken every 3–4 min for periods up to an hour, followed by a turn-round of 10–15 min for cooling, before returning. Such services operate with d.c. vehicles for 18–20 hours per day at present. A cooling system able to maintain the rectifier operating temperature on such a service within required limits would need a large capacity, with consequent increase in unit size and complexity. Difficulties arise with the question of whence to draw the cooling air, for filters clogged by the mixture of brake-shoe dust and track dirt would give reduced cooling, rectifier overheating and breakdown. The occurrence of temporary overloads and fault conditions must also be allowed for in the thermal rating.

Will mechanical vibration affect the germanium rectifier, for the arduous service to which the electrical equipment of a multiple-unit train is subjected by track vibration and shocks is an important factor?

From Section 3.1 I note that the impurity content of a germanium rectifier must be about 1 part in $10^9$. Have the manufacturing techniques reached the stage where this purity can be guaranteed? If not, difficulties will arise with unequal load sharing in parallel connection, owing to different electrical characteristics and consequent differential temperature rises. From a maintenance angle, would a replacement unit have exactly the same characteristics as that which has been removed?

**Mr. B. D. Ripley** (*at Newcastle upon Tyne*): Since considerable wastage must occur when sawing single-crystal germanium into wafers and pellets, and the recovery of germanium from the sludge is costly, I wonder whether a method of crystal cleava

has not been applied, since subsequent etching could restore the crystal boundary.

Am I correct in thinking that the difference between the theoretical and practical characteristic in the reverse direction is due to thermal agitation only?

The example quoted in the paper regarding the maximum allowable inverse voltage gives a factor of safety of about 2·4, but applies to a low-power rectifier. Is the factor of safety for the high-power rectifier of a similar order?

Has the crystal rectifier an application in supplying the excitation of large synchronous generators, the main requirements being continuity of supply and automatic voltage control? The former requirement has presumably been met in the electrolytic applications mentioned in the paper, but the solutions offered for the latter requirement are inelegant. Can we look forward to the development of the triggered diode and power transistor in the near or remote future?

**Mr. D. B. Corbyn** (*at Stafford*): The successful application of both types of rectifier depends mainly on cooling and control methods, and while air cooling is best for silicon rectifiers, there is much advantage in water cooling for large-area germanium cells. There appears to be some difficulty in transferring much heat to cooling fins in air. Is this found in practice?

Water cooling will give a smaller rectifier volume and small cheap connections of particular value for heavy currents. The heat is rejected by a heat exchanger, specially designed for its purpose and without electrical complications, placed in a convenient position, not necessarily in the rectifier. Overload capacity is provided by the cooling water. In the tropics one can nearly always find water considerably below the day-time temperature.

Control methods will be compared with those used for selenium rectifiers, and there is an enormous disparity in the rectifier self-capacitance in the two cases. Some 400 cm² of selenium would replace 1 cm² of germanium at a mean current of, say, 100 amp. The capacitance of selenium is about $0\cdot02\,\mu F/cm^2$ and hence the total capacitance is about $8\,\mu F$. How does this compare with the added capacitance described by the authors?

There appears to be increased risk of breakdown if high reverse voltage is applied immediately conduction ceases. Does this cause difficulty, and can germanium rectifiers be used in blocking circuits?

In auto-self-excited-transductor control the effect is similar to that of ignition delay in a mercury-arc rectifier and a large reverse voltage is suddenly applied at the end of conduction. Is this harmful?

A cinema rectifier using a series condenser to provide ballast impedance instead of the more common choke ballast has recently been announced. Resistance switching was employed, and I should be interested to know why this was necessary.

**Mr. T. J. Rowlands** (*at Stafford*): During short-circuit directly across the d.c. terminals of 3-phase bridge equipments, the direct short-circuit current is composed of peaks and troughs which subside after a few cycles when the fault current settles down to a sensibly steady value; the peaks occur when a rectifier conducts a major loop of current, and the troughs when it conducts a minor loop of current. By increasing the reactance/resistance ratio of the installation, the asymmetry of the fault current over the first few cycles becomes more pronounced, so that 15% reactance in an equipment having 2% resistance produces a peak d.c. short-circuit current of approximately 12 times the normal current, the steady short-circuit current being about 7½ times normal. In the 3-phase bridge connection, one rectifier arm has to carry all this peak direct current during a conduction period. Because of the asymmetry, the overall duration of conduction is prolonged, and the heat produced in a rectifier subjected to the initial asymmetrical loop will be double that produced by the ensuing symmetrical loops, and about 50% greater than that produced by a symmetrical loop having a peak value 10 times normal.

Because of these phenomena, the fault-current tests are not adequately defined unless reference is made to the pulse duration as well as its peak value. The disproportionate reduction of fault currents over the first critical cycles, obtained by adding reactance, is an important factor in favour of derating rectifier units and simultaneously avoiding low power factors and high regulation. Can the authors express their opinion on this?

**Dr. R. Feinberg** also contributed to the discussion at Manchester, and **Messrs. D. L. Smart** and **J. J. L. Weaver** to the discussion at Stafford.

**Messrs. T. H. Kinman, G. A. Carrick, R. G. Hibberd** and **A. J. Blundell** (*in reply*): In answer to Dr. Read, the hole-storage current prolongs the period of overlap; during this time energy is stored in the inductance of the two overlapping phases, and at the end of it part of this energy appears as a momentary increase in direct voltage and part is absorbed in oscillations. On balance there is a slight reduction of direct voltage and power factor. Since the flow of current in a germanium or silicon device involves the setting up of non-equilibrium densities of holes and electrons, any change of conditions will cause a transient condition, both in the forward and inverse directions of current flow. When forward voltage is suddenly applied to a germanium diode there is a hole-shortage pause before the current attains its steady value; this is not a condition which occurs significantly in a normal power rectifier.

The curve shown in Fig. 23 differs from that given by Miller owing to a difference in the value of $\alpha_i$, the ionization rate. We have obtained rectifier characteristics which fall into several categories, each of which calls for slightly different mechanisms or constant values to account for them.

Mr. Polgreen suggests that silicon will play a major part in semi-conductor applications. At present we can say only that germanium appears to be preferable for low-voltage applications and that silicon appears to be necessary for high-temperature conditions. It is certain that the production of pure single crystals of both silicon and germanium will become much easier and less costly.

The hopes raised by selenium and copper oxide that a non-ageing rectifier has been found seem likely to be fulfilled by germanium and silicon, whose forward characteristic appears very constant even under adverse conditions, and the reverse characteristic also constant with hermetic enclosure.

The commutator is likely to become obsolete as a means for generating direct current on a large scale, but a long time may elapse before the advent of a semi-conductor device with a rotating shaft delivering power.

Mr. McBreen inquires about fault-current conditions. Present-day cells will withstand eight times full-load current for the clearance time of a medium-speed d.c. circuit-breaker. The large regulation caused by high transformer reactance is in some cases unacceptable, and must then be avoided by suitable transformer connections or by derating.

Care must be taken that the cooling of all cells in an equipment is reasonably equal; no derating to allow for differences of cooling is then necessary.

Mr. Whyman discusses the application to locomotives and motor-coaches operating from an a.c. supply. In hot countries silicon will have an advantage for this duty, but in Britain germanium is quite suitable, and a motor-coach fitted with a compact germanium rectifier operating from a 6·6 kV 50 c/s overhead line has been in regular passenger service on the Lancaster–Heysham line for several months. The hole-storage effect is

normally not so large as shown in Fig. 15, and with suitable capacitors fitted has been found to produce no detectable interference. The comparison with a multi-anode rectifier is somewhat misleading and was made only because rectifiers of that type have been in experimental service on the same line.

In answer to Mr. Shepherd, in power rectifiers resistivity is of indirect importance, since it is usually correlated with the total donor impurities which determine the width of the space-charge region and hence the electric field. The limitation of inverse voltage with a given electric field is a surface rather than a bulk phenomenon, and it is found that a germanium surface which is not in an absolutely perfect condition tends to initiate a destructive breakdown above about 100° C. In silicon, junction temperatures of 250–300° C can be used and the surface does not appear to be so limiting. A maximum junction temperature having been decided upon, the peak inverse voltage must be determined from thermal-stability considerations in the maximum junction temperature region.

Mr. Kelsall asks about the use of germanium rectifiers in magnetic amplifiers. The ideal of zero forward resistance and zero reverse current is approached much more closely with germanium and silicon rectifiers than with previous types. The hole and electron theory is applicable to selenium and copper-oxide rectifiers, but with less exactitude because these materials are polycrystalline. The phenomenon of diffusion of minority carriers, which is important in germanium and silicon, does not play a very important part in the theory of selenium and copper-oxide units, which act as barrier-layer rectifiers. Other inter-metallic and non-metallic compounds can theoretically be used as the basis of rectifiers, but have properties which make their practical application difficult at present.

In answer to Mr. Hanson, germanium and silicon power rectifiers operate satisfactorily up to at least 1 kc/s, and smaller current devices have been made to operate at high radio frequencies. Silicon rectifiers having a rating of several kilowatts per cell have been produced.

We agree with Mr. Robson that some derating of cells is needed in a large equipment, owing to thermal and electrical irregularities. The rating per cell in such an equipment can be as much as 80–90% of its rating when used in a 3-phase bridge of six cells.

Mr. Davison discusses the cooling of traction rectifiers mounted on coaches. The small power loss of germanium and silicon rectifiers makes air cooling simple, even for continuous operation at full load. The dirt in the cooling air is not expected to cause difficulty, since other types of rectifier which have cooling-air passages of similar size operate satisfactorily in industrial locations where the air is equally dirty. The individual rectifier cells are so small and light that they are unaffected by the low-frequency vibration experienced on a sprung vehicle. Manufacturing methods to-day produce cells which have characteristics sufficiently close to avoid complicated matching.

Mr. Ripley suggests the production of wafers by crystal cleavage; this would be likely to distort the crystal structure and make the wafers unusable. A complete theoretical description of the inverse direction would involve many factors; the saturation component is affected by thermal generation in traps on dislocations, injection and generation at the back contact and generation at the germanium-air surface. Another component may be due to channel conduction in $p$-type layers which reach from the indium to the back contact. There would be a voltage-dependent component due to generation within the space-charge region and at the portion of the surface within the space-charge width. The breakdown region of the characteristic will be due to avalanche phenomena in the space charge and across the surface of the barrier. If the wafer is thin, punch-through may occur when the voltage is such that the space charge extends right through the wafer. Combinations of these effects, together with the possibility of a stable arc appearing across the surface of the barrier, account for almost all of the observed characteristics.

The safety factor on inverse voltage is of the order of 2 for a high-power rectifier. The use of germanium and silicon rectifiers for supplying excitation current to synchronous machines is an ideal application.

Both Mr. Ripley and Mr. Weaver ask about power transistors and trigger diodes. The development of germanium power transistors is encouraging; units have already been made to switch about half a kilowatt in the laboratory. The extension of the trigger diode (double-base diode) to powers greater than about 10 watts appears to be difficult and little progress has been quoted.

Mr. Rowlands and Mr. Smart suggest that high reactance is not a good method of limiting d.c. short-circuit current, owing to the asymmetry of the input current to the rectifier during short-circuit. The total effective resistance of an equipment is seldom lower than 3%, so that the asymmetry is usually less than the figures quoted. However, a rectifier cannot fail during complete short-circuit, since the inverse voltage across it is then zero; during a limited short-circuit the asymmetry does not occur and high reactance is of benefit. We think that a high-speed short-circuiting switch to protect the rectifier against fault currents is an undesirable complication.

Mr. Corbyn advocates water cooling for large-area germanium cells. In certain cases, particularly in the tropics or on low-voltage rectifiers, direct water cooling is preferable; but, in general, we feel that the problems involved in the supply of clean non-corrosive water with adequate protection against interruption and prevention of leakage, make air cooling preferable. The self-capacitance quoted for selenium rectifiers would be of the right order to prevent hole-storage voltage spikes, but the lower purity and polycrystalline nature of selenium prevent any appreciable hole-storage current. Germanium rectifiers are very suitable for blocking circuits and transductors; capacitors may be required if there is a possibility of rapid voltage reversal. In the germanium cinema rectifier, resistance switching is used to avoid circuit resonance at no load.

# DISCUSSION ON

# 'THE INSTALLATION OF METAL-SHEATHED CABLES ON SPACED SUPPORTS'*

*Before the* South Midland Supply and Utilization Group *at* Birmingham 10*th October, the* North-Western Utilization Group *at* Manchester 11*th October*, 1955, *and the* Mersey and North Wales Centre *at* Liverpool, 9*th January*, 1956.

**Mr. H. M. Fricke** (*at Birmingham*): For cables supported on concrete posts by the side of railway lines the variation between minimum winter and maximum summer temperatures can presumably exceed 100° F. Buried cables would be at more even temperatures and would also be less subject to damage by derailment.

I would stress the advantage of aluminium as a sheathing material where some measure of strength is a useful feature.

**Mr. M. L. Gorham** (*at Birmingham*): The author makes no mention of the method for dealing with straight joints. Does he recommend that any special precautions should be taken at such positions?

**Mr. J. R. Anderson** (*at Birmingham*): Have failures occurred with cables in buildings, where architects frequently require the cables to be installed in the heating ducts? Under such circumstances it is probable that the maximum ambient temperature would coincide with the maximum loading of the cables.

**Mr. R. A. Joseph** (*at Birmingham*): Has the interpretation of some of the results been rendered difficult by the presence of corrosion resulting from atmospheric conditions?

There seems to be no case for the use of cleat-type supports, except when three cables are run together or are in vertical situations.

**Mr. H. J. Gibson** (*at Birmingham*): The expansion and contraction of single-core cables due to cyclic variation of load is understandably fairly simple compared with that of 3-core cables, where the lay of the cores produces a tendency to twist. This is very apparent when 3-core cables drawn into ducts are subject to cyclic temperature variations and frequently this causes one end of a long length to twist through 90° as well as forcing the joint out of level, giving rise to cracked joints, wipes and subsequent failures. I have known of at least one case where a joint was twisted 180° as well as being forced up at a steep angle in the road box. Similar twisting in supports would, I think, give rise to different strains and different movements from those which would be purely longitudinal with the single-core cable.

**Mr. G. S. Buckingham** (*at Birmingham*): I support the author's economies and would always specify his method for the erection of railway cables above ground.

We are now arranging to load our cables more fully than hitherto and may therefore expect them to reach a conductor temperature of 80° C. If the initial sag between supports is 2%, what additional sag might be expected with a cable running fully loaded? If two cables were erected on the same supports, would there be an obvious unevenness if one cable were fully loaded and the other unloaded?

The wider spacing of supports recommended by the author will result in additional bearing pressure between the cable and the support and will also add to the load on the supporting structure; these facts must be borne in mind in any new installation.

**Mr. T. G. D. Wintle** (*at Birmingham*): What are the practical limits to which the amount of sag can be controlled during the laying of the cable?

I imagine that loops of cable would tend to increase because of a type of ratchet action. Does this, in fact, occur?

* Holttum, W.: Paper No. 1814 U, April, 1955 (see 102 A, p. 729).

**Mr. E. L. Davey** (*at Manchester*): The author deals with lead-, lead-alloy- and aluminium-sheathed cables. Although in the former the bending or expansion generally involves strain beyond the elastic limit of the sheath, in both cases the cables have been treated as elastic beams; but in the former case practical work has been carried out to supply data relating to the bending because of the straining beyond the elastic limit.

I do not agree that the soil imposes a mechanical restraint on the cable expansion and that this is a factor which reduces the expansion of the sheath for buried cables. Another factor in the expansion equation is the size of the conductor, and generally speaking the larger the conductor the greater is the expansion duty on the sheath; this is accentuated if the cable is of the single-core instead of a multi-core type. Wire armouring of the cable is of great benefit in reducing the risk of sheath fatigue, owing to the continuous support given. Steel-tape armouring—not to be confused with steel-tape reinforcing for pressure cables—is to be deprecated, since selective expansion occurs in the butt spaces between the edges of the steel tape applied next to the sheath. I have seen such cables with pronounced sheath fatigue on direct-laid cables.

I agree with the author's proposals regarding the installation of cables on posts. In any lead- or lead-alloy-sheathed cable run above ground the basic aim is to ensure that the expansion effects occur uniformly or regularly, so that the strain on the sheath is limited as much as possible. The spacings which have been normally used are definitely low compared with the author's calculated values. One drawback of the increased spacings is that the cables are more susceptible to damage from external influences, such as weights, etc., than for shorter-span cables. At joint positions and wherever the cable run is offset it is important that steps are taken to prevent selective expansion. The plumbing of a joint sleeve to a cable sheath enlarges the grain size of the sheath and lowers the fatigue level, so that joints must be regarded as items to be protected, especially against fatigue.

I consider that cables should be sheathed only with lead alloys. From the aspect of vibration and flexing fatigue pure lead is suspect and no advantage exists, except a small price differential. From the fatigue aspect B-alloy is superior to E-alloy, but it is more difficult to extrude and involves a risk with the associated plumbing during jointing operations. If an excessive temperature is used, so-called 'hot short cracking' of the sheath may occur. In addition, the alloy is much stiffer than E-alloy and hence the cable is more difficult to install. C-alloy was superseded by E-alloy during the last war, and the latter is now to be preferred on account of slightly superior fatigue level and lower price. Of the other alloys, D-alloy, although possessing a higher fatigue level, is liable to fracture when subject to a combination of alternating stress and tension, and generally can be said to be lacking in ductility.

The quoted fatigue values of lead alloys are given as those stresses (based on a modulus of elasticity of $2 \cdot 5 \times 10^6$ lb/in$^2$) which correspond to the alternating strain at which failure occurs after $10^7$ cycles applied fairly rapidly. For fewer cycles the stress rises fairly rapidly, and a further point is that the fatigue value decreases with increase of temperature. It is also important to

note that the quoted value is given as a plus or minus amount, so that for the change of bend of a sheathed cable it is the change of stress or strain which is considered. A pure-lead-sheathed and armoured cable and a similar E-alloy-sheathed and armoured cable were subjected to flexing tests at a rate of approximately 1 cycle every 10 sec, the flexing consisting of a change of bending radius; the former developed a cracked sheath after 7 000 cycles, while the latter withstood 67 000 cycles. Experience with the transport of cables in ships, where vibration can occur, shows that the E-alloy cables possess great advantage over the pure-lead-sheathed cables. In my opinion, the case for the elimination of unalloyed lead sheaths is overwhelming.

The author's figures for aluminium-sheathed cables show that the spans should be very much greater than for lead-sheathed cable, because the sheath behaves as an elastic beam. This point should be emphasized strongly, for correct installation may eliminate risks of expansion troubles in these cables at later dates.

In Table 8 I cannot understand why the expansion on an unarmoured cable is less than that of either the conductor or lead sheath, and I am dubious of these figures in the absence of a satisfactory explanation for the anomaly.

The author indicates that very little trouble has been experienced with solid-type cables even after the sheaths have been fractured. Too much attention should not be paid to this, and the presence of a cracked sheath should be regarded as a serious matter.

Can the author correlate the fatigue cases he has recorded with the fatigues in the inner or membrane sheaths of compression cables where the sheaths are expanded and compressed during every temperature cycle?

**Mr. E. P. G. Thornton** (*at Manchester*): If a cable on brackets is laid without sag during the heat of the summer, and is subsequently loaded only very lightly during the winter, under very cold weather conditions the cable will contract and may severely damage the post run. Several cases of this have been experienced.

I do not agree that fatigue limits determined for comparatively rapid stress cycles cannot be regarded as conclusive evidence on the behaviour of various lead alloys under slow flexing conditions, and reports from British Non-Ferrous Metals Research Association clearly show that they can. Thus, for extreme conditions of flexing, B-alloy would be even more satisfactory than E-alloy, particularly since railway systems make extensive use of cables on post runs and vibration is almost always present in addition to flexing. Recent work in connection with the cross-Channel cable project clearly shows that an E-alloy-sheath cable is superior to a pure-lead-sheathed cable in its ability to withstand repeated flexing.

Would the author expect that the sheath life of an unarmoured cable would be greater if, on a given installation with bracket centres at only 50% of those he calculates as optimum, B.S. 7 or B.S. 480 unearthed cable were used instead of B.S. 480 earthed cable? Cables to former specifications would, of course, be stiffer, owing to the greater sheath diameter and thickness.

**Mr. T. E. Wilson** (*at Manchester*): Experience shows that bending and flexing of cables depends, not only on the span of the cable, but on other factors, such as weak ground or posts, the tension locked up in the armour wires and uneven spans. It is almost impossible to obtain spans of identical length, and even where the variation in span length is only small, it has been found that considerably more movement takes place in one long span between two shorter spans than in the adjoining spans.

In Section 3 the author points out that approximations and assumptions have to be made, and there would appear to be some divergence between calculated spans and those obtained on test, as shown in Section 16.2. Test spans are, in all cases, longer than the calculated span, in order to obtain the desired sag.

The results of the examinations shown in Section 6 of the paper are somewhat inconclusive, and it should be noted that in one case, where the ratio of the actual span to the calculated span is only 0·81, the behaviour of the cable is shown as satisfactory, whereas in two cases, where the ratios were 0·87 and 0·84 respectively, the cables are shown as unsatisfactory.

**Mr. J. Tozer** (*at Manchester*): It is not clear whether the cables in tunnels were subject to sunshine or whether the effects were brought about by cyclic loading only. If the latter, it would appear that there must be an enormous number of cable installations open to doubt.

The author shows how to evaluate the minimum distance between cable supports, but not the maximum distance which can be allowed. I should be grateful for his guidance in this respect.

Have the effects of subsidence been taken into consideration, for it is well known that subsidence has been prevalent on certain railway installations in this area?

**Mr. L. J. Archer** (*at Liverpool*): I agree that the author's methods are not observed in practice to any marked degree, and I think it desirable that those who prepare specifications for cable work should give this question serious consideration. A recent C.E.A. specification says that 'cable racks and supports should be such that the cables are supported in a neat and orderly manner and without undue sag'. This is obviously vague, because the question of what is undue sag has to be determined, but I suspect that those preparing this specification did not intend that supports should be as widely spaced as the author suggests.

Of the 17 or 18 cables which the author examined, only three had spacings which were anything like adequate according to his calculations, and it is significant that these are the ones whose behaviour has been entirely satisfactory; however, the number of cases investigated was rather small, and it is desirable that an investigation on a much wider scale should be carried out to check the author's mathematical conclusions.

Section 4 refers to supports closely embracing the cable, which I take to mean cleats which fit the cable tightly; yet the author says that towards the end of a run there may be difficulty in restraining the cable movement. I suspect that, where the cleats embraced the cable tightly and were placed fairly close together, a condition would be achieved which was similar to that which occurs when cables are laid in the ground and that there would be no greater strain at the end of the run than at any other point.

The author points out that the curvature of the cable decreases as the support spacing increases; this would not seem to be the case at the supports themselves, which might well be more critical than the curvature in the middle of the span.

The majority of cable users are not greatly concerned with vertical runs; these will generally not be higher than a transmission tower, where it is common practice to support the cable at 3 or 4 ft intervals, and I know of no trouble experienced in such cases. I suppose that with runs on colliery shafts the problem can become acute.

**Mr. A. V. Milton** (*at Liverpool*): When Table 1 is arranged in chronological order of the dates of installation it is obvious that in the nine installations up to 1929, the spacing of the supports was considerably greater than in (with one exception) the later installations. In the earlier period two items at 8 ft spacing exceed the author's calculated spacing and the shortest spacing was 63 in, the nominal/calculated ratio being 0·71. Leaving out item 16 in the nine later items, the spacings range from 54 to 42 in with nominal/calculated ratios ranging from 0·63 to 0·36

It is difficult to believe that it was a casual decision to space the supports at 8 ft in the earliest installations (items 11 and 7*b*) and it seems as if at that time the cable engineers had in mind principles the author is now propounding, based either o[n]

practical experience with earlier cables or on a technique used in other engineering fields—possibly for lead piping, which in many applications is subject to a wide range of temperature variation.

The installation of cables in or about buildings is subject to The Institution's Wiring Regulations, and here again the development through the varying editions is quite interesting. In the 9th edition (1928) the wording is 'the spacing shall be such as shall prevent appreciable sagging if fixed horizontally' without specifying any distance of the spacing. The 10th and 11th editions give the maximum spacing for cables up to 0·06 in on horizontal runs as 18 in, but if the cables are unlikely to be disturbed, a spacing of 10 ft for fixings is permitted. The 12th edition gives the same spacing up to 0·06 in, but where the cables are unlikely to be disturbed the maximum spacing is then 3 ft, and in the 13th edition (September, 1955) the spacing for cables up to 0·06 in is still 18 in, but for cables above 0·1 in$^2$ 'the manufacturers' recommendations shall be used'. Possibly the latter recommendation resulted from the researches that have culminated in the paper, but it would appear the manufacturers may have some trepidation in making any recommendations in view of the author's findings, which are so contrary to the accepted practice of recent years.

Cables in and about buildings are not under such arduous conditions as the majority of the examples in Table 1, which are on railway systems; they are not subject to the ambient-temperature variations and usually are sheltered from the sun; but in the majority of building cables the fracture of a lead sheath would be serious, because the lead sheaths are normally used in the protective earthing systems.

If further investigation of this subject is necessary before the cable makers can accept or refute the author's findings, it is to be hoped that there will be no delay in carrying out detailed inspections of many similar cable installations which have been in service for long periods, so that a wider range of data is available.

**Mr. J. E. Macfarlane** (*at Liverpool*): Inspection of the cable beside the railway track between London Bridge and Woolwich Arsenal showed that the supports are very much closer than postulated by the author. The cable seems to be covered with waves of material like gas tar, and no lead sheath is visible. Is there a hessian covering which has been treated with compound?

**Messrs. A. Shaw** and **P. Martin** also contributed to the discussion at Birmingham.

**Mr. W. Holttum** (*in reply*): I would preface this reply by further reference to the commonly held belief that the best method of installation of metal-sheathed cables is by burying in the ground. Fuller consideration of the matter has led me to the conclusion that installation on spaced supports, carried out to sound design, is the best method for such cables where it is not excluded by the nature of the route. The contrary view is due to the fear that the many advantages, in particular economy and accessibility, will be more than offset by the danger of sheath fracture caused by the strains and stresses arising from thermal expansion and contraction. In view of the capital values involved, it is clearly of the first importance that it should be known whether installation on spaced supports is to be regarded as an undesirable expedient or the preferable method, and it is hoped to make a further contribution to the subject later.

In reply to Mr. Gorham, conditions at joints are easier than on buried cables, since longitudinal stress is relieved. Joints should be supported where they are heavy enough to require it, and there should be a cable support near each end and then standard spans.

I have no information about cable failures in buildings, asked for by Mr. Anderson, but, under any conditions, loading should, of course, be such that the rated temperature is never exceeded.

I can assure Mr. Joseph that the investigation was not complicated by corrosion. Cleats are required only where there is a tendency to movement to be restrained.

Mr. Gibson's experience of joints in duct cables confirms my view that this method of installation is fundamentally unsound, and should never be employed where the alternative of spaced supports or burying is available. There is no reason to fear damage at the ends of supports, since the more pronounced the bend, the less is the movement.

I am glad that Mr. Buckingham favours the method advocated. From eqn. (12), an initial sag of 2% would increase to 2·54% for 0·06% expansion, which should provide for a conductor temperature of 80° C.

I cannot give Mr. Wintle practical limits for the control of sag, but there should be no difficulty in acquiring skill in sagging with any required degree of accuracy. With suitable spacing, there will be no longitudinal movement after any slight initial self-adjustment has taken place.

I do not share Mr. Davey's objection to steel-tape armour. While wire armour has the merit of possibly reducing movement by its much lower thermal expansion than conductor or sheath, single-core cables do without it, and I regard steel-tape armour, which gives excellent mechanical protection, as sound practice for supported 3-core cables. I am glad to see the measure of support by Messrs. Davey and Thornton to the use of alloy E for supported cables, but I would point out that the reason for the long spans required for aluminium-sheathed cables is the high modulus of elasticity of the sheath and not the fact that it behaves as an elastic beam. I agree that the expansion of the unarmoured cables shown in Table 8 is unaccountably small. The effect, however, was checked by double repetition in each case. Each cable had previously been tested under the restraint of wire armour, which may have some bearing, but I have not found an explanation. It had occurred to me that the cyclic circumferential strains in compression cable sheath would provide an interesting comparison, and I found that these amount to about 0·002 on the outer surface and 0·003 on the inner, in addition to the longitudinal repressed expansion. This is of the same order as the maximum cyclic longitudinal strain on the sheath of a supported cable, which may be taken as in the range 0·001–0·005. The compression-cable sheath, however, has the advantage of complete protection from oxygen.

Mr. Thornton comments on the effect of laying without sag in cold weather. The chief danger from this is pulled joints, and sag should, of course, be provided. I have examined the results of researches on the effects of rapid and slow straining of lead alloys, but cannot agree that a relation between them has been established. Regarding the use of B.S. 7 or unearthed cable, if the dimensions of cable, span, and sag were such that the sheath strain was reduced, the life should be increased. Whether the greater thickness would itself contribute to this I cannot say.

In reply to Mr. Wilson, cyclic thermal strain is more important than one-way movement due to weak ground or posts. I also have seen concentrated bending in spans only slightly longer than the rest, but this is the consequence of too-short spans. The purpose of the tests discussed in Section 16.2 was to obtain rough confirmation of theory, and it was to be expected that longer spans would result, since it was an accelerated test. Regarding consistency of span length ratios and behaviour, it should be noted that item 16 in Table 1 was only lightly loaded, and would probably behave badly with severer operating conditions.

The cables in tunnels were subject to ambient and load effects only. There is, as suggested by Mr. Tozer, a large majority of supported cable open to doubt. Mr. Tozer refers to the important question of maximum permissible support spacing.

This is briefly discussed in Section 8.1, but there is no short answer, and it is hoped to go into the matter more fully in a future contribution.

The C.E.A. specification mentioned by Mr. Archer illustrates how appearance has been allowed to take precedence over engineering considerations. Nothing would, in my view, be gained by further investigation, unless it helped to convince the sceptical, which seems unlikely. Confirmation, if required, can best be obtained from installations designed on the principles advocated. The argument in Section 4 is based upon cleats which closely embrace but do not grip. If they grip and are close enough together the effect would, as suggested by Mr. Archer, be similar to the condition of being laid in the ground, except that end effects would be more likely. I agree that very wide spacing will cause sharper bending at supports, but it is the change of curvature with expansion that matters, and this might well be reduced.

The tendency to shorter spans in later installations, pointed out by Mr. Milton, must, I think, be assumed to be due to increased emphasis on appearance, the engineering aspect not being appreciated. The statement in The Institution's Wiring Regulations that the manufacturer's recommendations for support spacing should be used is not to be ascribed to the work of the paper, and, in fact, I believe that this part of the Regulations needs further attention.*

Railway engineers have carried out a great deal of maintenance of cable finishes by hand taping and compounding, and this is doubtless the cause of the appearance Mr. McFarlane has observed.

In the London discussion, Mr. E. A. Cullen drew attention to some cables installed with spans short for their size, on which he had found no sign of trouble in a long association until he left the area. I thought this an interesting case and was able to arrange an inspection, and found that a large number of places showing compound leakage had since developed, some of them quite severe.

* HOLTTUM, W.: 'Spacing of Supports for Metal-Sheathed Cables', *Electrical Contractor and Retailer*, February, 1956.

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*Example.*—SMITH, J.: 'Overhead Transmission Systems', *Proceedings I.E.E.*, Paper No. 3001 S, December, 1954 (102 A, p. 1234).




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